

This thesis deals with dynamical and arithmetic properties of substitutive sequences. A major theme explored here is the belief that object coming from substitutions of independent arithmetic origins should have little to no common structure. A prototypical example of such a result is Cobham's theorem concerning automatic sequences.

In our first result we show a finitary version of Cobham's theorem, which provides a complete characterisation of the sets of words that can appear as common factors of two automatic sequences defined over multiplicatively independent bases. This generalises the classical result of Cobham and answers a question posed by Jeffrey Shallit. The proof is effective and gives an algorithm which given two independent automatic sequences  $x$  and  $y$  computes the set  $\mathcal{L}(x) \cap \mathcal{L}(y)$  of their common factors.

In our second result we look at a problem of how to recognize that a fixed point of a (nonconstant length) substitution is automatic raised recently by Allouche, Dekking and Queffélec. We give a complete solution to this problem in terms of the growth properties of the substitution: in the spirit of Cobham's Theorem we show that this can happen only if the original substitution is close to being of constant length itself.

To prove the above results we study the dynamical properties of substitutive sequences. We show that transitive subsystems of substitutive (resp., automatic) systems are substitutive (resp., automatic) and give a simple characterisation of all automatic sequences in a given automatic system in terms of the so-called quasi-fixed points of an underlying substitution. We also obtain a quantitative version of recognizability for automatic sequences.