

## Four-nucleon force contribution to the binding energy of $^4\text{He}$

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**Abstract.** We study the four-nucleon force contribution to the binding energy of  $^4\text{He}$  in the framework of chiral nuclear interactions. The four-nucleon forces start to contribute in the next-to-next-to-leading order. We discuss our power counting expectations for the size of the 4N contribution and then explicitly calculate it in first order perturbation theory. Our expectations agree with the results. Quantitatively, the contribution might be significant. This motivates further studies in more complex nuclei.

### 1 Introduction

One of the main goals of nuclear physics is the understanding of the properties of nuclei based on nuclear interactions. It is generally accepted that the Hamiltonian for a nuclear system is driven by nucleon-nucleon (NN) pair interactions, for which highly accurate models have been developed [1–3]. But the application of these models to light nuclei [4–8] has shown that NN interactions alone are not able to provide a sufficiently accurate description of the data. This led to the conclusion that three-nucleon forces (3NF's) are required to describe nuclei based on microscopic interactions<sup>1</sup>. Models for three-nucleon (3N) interactions exist starting with the venerable Fujita-Miyazawa force [9]. Such models have been refined by repulsive short distance pieces [10] or implementing constraints by  $\pi$ -nucleon ( $\pi$ N) scattering [11,12]. Unfortunately, due to the phenomenological character of NN interactions, none of these models is based on a common footing with any of the modern accurate NN interactions<sup>2</sup>. This is however a basic requirement of any combination of NN and 3N forces since both cannot be defined independently of each other [15]. Nevertheless, brute-force combinations of such NN and 3N force models, that are tuned to at least describe the  $^3\text{H}$  binding energy, give quite reasonable results for 3N scattering observables [16] and binding energies of light

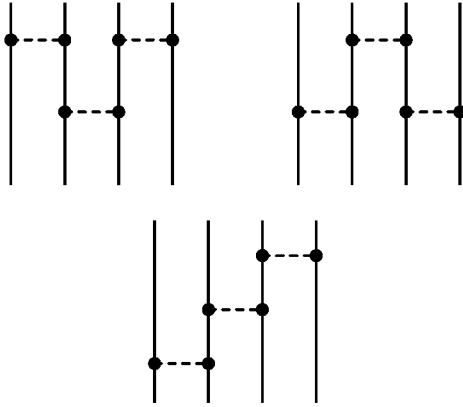
nuclei [4–6,17,8]. But at the same time, such results show deficiencies that indicate that part of the nuclear Hamiltonian is not understood sufficiently well.

A systematic scheme to derive the nuclear Hamiltonian is based on chiral perturbation theory (ChPT). Here the approximate but spontaneously broken chiral symmetry of the QCD Lagrangian is implemented in an effective field theory in terms of nucleon and pion fields. Chiral symmetry constrains the possible couplings of these fields, especially for the pions being the pseudo-Goldstone bosons related to the spontaneous symmetry breaking. Due to these constraints, the Lagrangian and all diagrams can then be expanded in terms of  $\frac{Q}{\Lambda_\chi}$ , where  $Q$  is a typical momentum of the considered process or the pion mass and  $\Lambda_\chi$  is the chiral symmetry breaking scale of the order of the  $\rho$  meson or nucleon mass. For low momenta and systems with nucleon number  $A = 0$  or 1, this leads to a perturbative expansion of the relevant amplitudes. For  $A \geq 2$ , this expansion cannot be perturbative, since bound states (the nuclei) exist. Weinberg recognized that diagrams with purely nucleonic intermediate states, so called reducible diagrams, are responsible for this non-perturbativeness. He therefore suggested to expand a potential (the sum of all irreducible diagrams) using the standard power counting of ChPT. Reducible diagrams can then be summed up to infinite order by solving the Schrödinger equation based on such a potential [18,19]. This explains naturally why NN forces are driving the nucleon Hamiltonian and more-and-more-nucleon interactions become less-and-less important. It also enables us to derive NN and more-nucleon interactions within the same framework.

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<sup>1</sup> The term microscopic interactions refers to interactions among nucleons as basic constituents of nuclei.

<sup>2</sup> For an attempt to derive NN and 3N forces from a unified approach see [13,14].



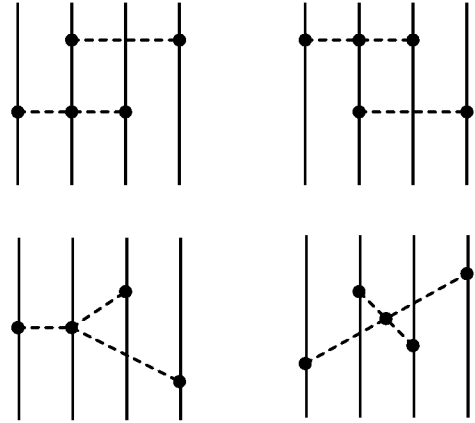
**Fig. 1.** Non-zero contributions to class I of 4NF diagrams  $\propto g_A^6$ .

It turns out that leading order (LO,  $Q^0$ ), next-to-leading order (NLO,  $Q^2$ ), next-to-next-to-leading order (N<sup>2</sup>LO,  $Q^4$ ) and next-to-next-to-next-to-leading order (N<sup>3</sup>LO,  $Q^6$ ) terms of the chiral expansions are required to obtain NN interactions that have an accuracy comparable to the modern phenomenological ones [20, 21]. The leading 3NF's appear in N<sup>2</sup>LO [22, 23], some parts of the subleading terms have been formulated [24] but not applied yet. The leading four-nucleon force (4NF) is of order  $Q^4$  and has been derived in [25, 26]. In these proceedings we report on the application of this 4NF, namely a calculation of its contribution to the binding energy of <sup>4</sup>He. This work goes beyond our first estimate of this 4NF contribution [27], since we now take the <sup>4</sup>He wave function in its full complexity into account. This is especially important for a reliable estimate of short range contributions of the 4NF.

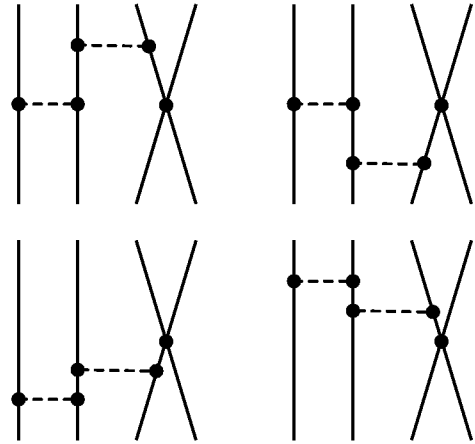
We start introducing the chiral 4NF in Sec. 2. Then we turn to the more technical aspects and define the ingredients of the actual calculations in Sec. 3. In Sec. 4, we discuss our power counting expectations for the size of the 4NF. The numerical results have been obtained using a Monte-Carlo approach that is introduced in Sec. 5. The results are given in Sec. 6, which leads us to the conclusions and the outlook in the final section.

## 2 Chiral interactions and 4NF's

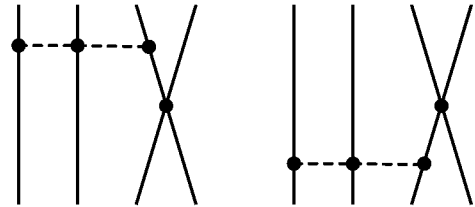
Four-nucleon interactions have already been discussed in the 1980's [28, 29]. At the time, the conclusion was that the contribution is probably small enough to be neglected. Given that 3NF's were known to be much more important but much less understood at the time than today, it was reasonable to neglect the 4NF's based on the results obtained. But it is timely to reconsider this part of the interactions now for two reasons. Firstly, we are now in position that much more accurate nuclear structure calculations are possible, the aim being to predict the masses even of drip line nuclei. For such an endeavor, the accuracy of the underlying forces needs to be much higher and 4NF's might become quantitatively important. Secondly, we have now a systematic scheme to derive consistent NN, 3N and 4N interactions based on chiral perturbation theory. Therefore,



**Fig. 2.** Non-zero contributions to class II of 4NF diagrams  $\propto g_A^4$ .



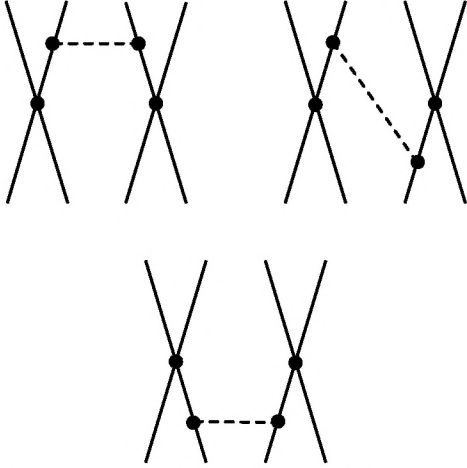
**Fig. 3.** Non-zero contributions to class IV of 4NF diagrams  $\propto g_A^4 C_T$ .



**Fig. 4.** Non-zero contributions to class V of 4NF diagrams  $\propto g_A^2 C_T$ .

we are now in the position to derive the complete leading contribution of 4NF's consistently to the chiral NN and 3N interactions.

Here, we restrict ourselves to the effective theory without explicit  $\Delta$  isobar degrees of freedom. We stress that due to the strong coupling of the  $\pi$ N system to the  $\Delta$  and the small difference of the nucleon and  $\Delta$  mass, the inclusion of  $\Delta$  might be advisable. For the NN and for 3NF's, this has been done already (see e.g. [30–33]). The results confirm the importance of  $\Delta$ 's in nuclear interactions. For the 4NF, the contribution due to  $\Delta$ 's was estimated in [34] based on a phenomenological approach. This study indicated that the  $\Delta$  contribution to 4NF's is small. It will be interesting to look at this estimate again based on chiral

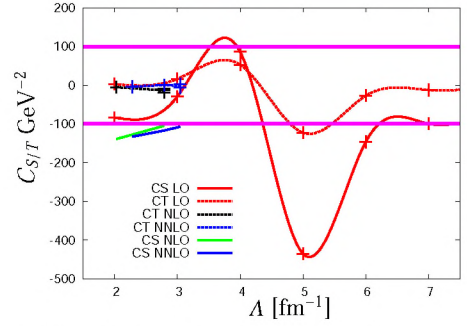


**Fig. 5.** Non-zero contributions to class VII of 4NF diagrams  $\propto g_A^2 C_T^2$ .

interactions and to confirm that this conclusion persists independent of the choice for cutoffs, but this is beyond the scope of this study.

Neglecting  $\mathcal{A}$ 's and based on the power counting of Weinberg, one only gets NN interactions in LO ( $Q^0$ ) and NLO ( $Q^2$ ) and additional NN interactions and first 3NF's in  $N^2$ LO ( $Q^3$ ). Up to this order the complete nuclear Hamiltonian has been derived. The  $Q^4$  terms ( $N^3$ LO) have been completely formulated for the NN force and they proved to be quantitatively important for an accurate description of NN data [20,21] up to the pion production threshold. At this point, parts of the subleading 3NF's have been formulated [24], but they have not been applied yet. The approach has been reviewed in [35,36]. For the explicit expressions of the NN and 3N forces, we refer to Ref. [37] where all results have been derived in the same scheme of unitary transformations that was used for the 4NF's applied here. The approach is well suited to end up with standard nuclear potentials that can be directly applied to few-nucleon systems since all interactions are manifestly energy-independent. Since the same approach was used for the NN and 4N force, it is insured that both are consistent to each other.

Additionally to the  $Q^4$  NN and 3NF's, there are also first 4NF's in this order. The derivation of the complete set of these terms has been done in Refs. [25,26] and showed that the leading 4NF does not only consist of pion exchange pieces, but also of short range pieces that are directly linked to corresponding short range pieces of the NN interaction. It is useful to classify the contributions according to their dependence on the axial-vector coupling constant  $g_A$  and the low energy constants (LEC's)  $C_T$ . In Refs. [25,26] eight classes have been identified. Some of the contributions are zero, therefore only class I ( $\propto g_A^6$ ), class II ( $\propto g_A^4$ ), class IV ( $\propto g_A^4 C_T$ ), class V ( $\propto g_A^2 C_T$ ), and class VII ( $\propto g_A^2 C_T^2$ ) terms have to be considered. In Figs. 1 to 5, we summarize the topologies of the diagrams contributing to the 4NF. The diagrams shown visualize expressions that have been derived algebraically. Note that some of the diagrams look as if they are reducible itera-



**Fig. 6.** Cutoff dependence of the LO LEC's  $C_S$  and  $C_T$  for various orders of the chiral interaction.

tions of NN or 3N interactions. We however only consider the irreducible parts here, which naturally separate in the expressions derived. To arrive at the final expressions, it is also mandatory to study 3NF's consistently. It turns out that the requirement that 3NF's are renormalizable further constrains the expressions for the 4NF's. For details, we refer to Ref. [26] where also the final expressions of the 4NF can be found.

As mentioned earlier, there is a relation of the short range part of the leading 4N and LO NN interactions. The LO NN interaction consists of the  $1\pi$ -exchange and two contact interactions

$$V_{LO} = - \left( \frac{g_A}{2F_\pi} \right)^2 \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{q^2 + m_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + C_S + C_T \sigma_1 \cdot \sigma_2.$$

Here,  $q$  is the momentum transfer from one nucleon to the other and  $\sigma_i$  ( $\tau_i$ ) are Pauli matrices acting in spin (isospin) space of nucleon  $i$ . The strength of the  $1\pi$  exchange is determined by the axial-vector coupling constant  $g_A$  and the pion decay constant  $F_\pi = 92.4$  MeV. The strength of the contact terms is parameterized by the LEC's  $C_S$  and  $C_T$ , which are determined by a fit to NN scattering data and/or the deuteron properties. Interestingly, parts of the 4NF depend on  $C_T$  stressing the strong relation of NN and 4NF's and the need for consistent combinations of NN and more-nucleon interactions.

Fortunately, the strength of the 4NF is completely determined by LEC's that also appear in the leading NN interaction. Our estimate below will therefore be completely parameter independent.

The nuclear interaction needs to be regularized in order to obtain a well-defined Schrödinger equation. This is usually done by multiplying the potential matrix elements with a cutoff functions, e.g. exponentials

$$V(\mathbf{p}', \mathbf{p}) \longrightarrow \exp\left(-\left(\frac{p'}{\Lambda}\right)^n\right) V(\mathbf{p}', \mathbf{p}) \exp\left(-\left(\frac{p}{\Lambda}\right)^n\right).$$

The cutoff functions dependent on the relative momenta of the nucleons (here  $\mathbf{p}$  and  $\mathbf{p}'$ ) and a cutoff  $\Lambda$ . The power  $n$  is usually chosen between 4 and 8. Below we will show leading order results for  $\Lambda = 2$  to  $7$  fm<sup>-1</sup> and higher order results for  $\Lambda = 500$  to  $600$  MeV.

As can be seen from Fig. 6, the  $C_S$  and  $C_T$  are strongly cutoff dependent. From naturalness, one would expect that



**Table 1.** BE's  $E(^3\text{He})$  and  $E(^4\text{He})$  for  $^3\text{He}$  and  $^4\text{He}$  for selected phenomenological models and LO, NLO and  $\text{N}^2\text{LO}$  chiral interactions compared to experiment. For chiral interactions, the cut-off dependence is indicated given the minimal and maximal binding energy obtained in our calculations. All energies and the cut-offs are given in MeV.

interaction	$E(^3\text{He})$	$E(^4\text{He})$
AV18+Urbana-IX	-7.72	-28.5
CD-Bonn+TM99	-7.74	-28.4
LO	-5.4 ... -11.0	-15.1 ... -39.9
NLO	-6.99 ... -7.70	-24.4 ... -28.8
$\text{N}^2\text{LO}$	-7.72 ... -7.81	-27.7 ... -28.6
Expt.	-7.72	-28.3

$C_S$  and  $C_T$  are of the order of  $100 \text{ GeV}^{-2}$ . For  $C_S$ , this naturalness estimate holds for most cutoffs and orders with a few exceptions. Such exceptions can be linked to the appearance of spurious bound states in the NN system [38]. Generally, the  $C_{S/T}$  for such  $\Lambda$  can be large, but their contribution to interactions are nevertheless natural, since the short distance wave function is suppressed for such  $\Lambda$ . Contrarily,  $C_T$  is much smaller than the naturalness estimate. This has been observed already in [39] and can be traced back to the approximate Wigner symmetry of nuclear interactions. It will be interesting to study the importance of terms of the 4NF proportional to  $C_T$  below.

### 3 $^4\text{He}$ wave functions

We are going to estimate the 4NF contribution in first order perturbation theory. For such an estimate the expectation value of the relevant operators with respect to the  $^4\text{He}$  wave function has to be calculated. Therefore, we would like to summarize briefly which wave functions enter our calculations.

Although consistent results can only be obtained based on chiral nuclear interactions, we have also performed calculations based on the modern phenomenological interactions AV18 [1] and CD-Bonn [2]. In both cases, we augment the NN interaction by phenomenological 3NF's based on  $2\pi$  exchange, which have been adjusted to the  $^3\text{H}$  binding energy and, for Urbana, also to nuclear matter density. For details on this adjustment see Refs. [5, 40].

For studying the cutoff dependence of the expectation values, it is also useful to study the 4NF for the leading order wave functions. Here, we follow the scheme of Ref. [38]. However, we only consider  $s$ -wave interactions, so that only two LEC's need to be adjusted, which we fit to the deuteron binding energy and the  $^1\text{S}_0$  phase shift at 1 MeV laboratory energy.

Our most consistent calculations are based on chiral nuclear interactions of order NLO and  $\text{N}^2\text{LO}$ . Here, we apply the interactions of Ref. [21] which have been derived in the same framework as the 4NF. This guarantees consistency of both parts of the interaction. In order  $\text{N}^2\text{LO}$  also the leading 3NF's are included. The relevant LEC's have

been adjusted to the  $^3\text{H}$  binding energy and the nd doublet scattering length as outlined in [23].

In order to obtain the wave functions, we solve Yakubovsky equations in momentum space in a partial wave basis [5]. For the representation of the wave function, we take angular momenta up to  $l = 6$  into account. This requires a large number of partial wave channels of the order of 1000 for the representation of the wave functions. For the Monte Carlo scheme described below, we need to transform the wave function from this partial wave basis to a basis depending on three-momenta and individual nucleon spin/isospin projections. This transformation has been implemented quite efficiently, still it takes the bulk of the computational resources.

We summarize the binding energy results in Table 1. One can see that the binding energies are well described for the phenomenological and chiral  $\text{N}^2\text{LO}$  interactions. Due to the correlation of the  $^3\text{H}$ ,  $^3\text{He}$  and  $^4\text{He}$  binding energies, this is not surprising. At LO and NLO, the binding energy of  $^3\text{H}$  cannot be adjusted, so that the dependence is still rather strong. The chiral expansion of binding energies is generally slowly converging, since the cancelation of kinetic and potential energy enhances small contributions to the interaction.

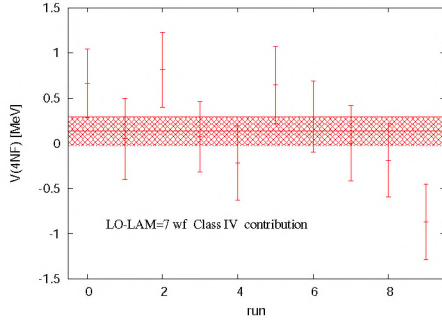
### 4 Power counting estimate

Before actually doing an explicit calculations for the 4NF, we would like to estimate its contribution based on general power counting arguments and previous experience. Although potential energies are no observables, it is useful to estimate higher order contributions to the binding energy based on the expectation values of the NN potential, since the chiral expansion is performed for this potential. Since typical momenta in nuclei are of the order of the pion mass, the small scale of the expansion is usually assumed to be of this order. There are some discussions on the large chiral symmetry breaking scale. Whereas for purely pionic processes, loop contributions can be well estimated assuming  $\Lambda_\chi \approx 4\pi f_\pi \approx 1 \text{ GeV}$ , this is probably not a valid choice for processes involving nucleons. Pion production is not explicitly included into the chiral interactions and the momentum scale associated with such processes is of the order of 400 MeV. At the same time, the momentum cutoff of higher order chiral interactions is in the same order of magnitude. Therefore, we use this value for our estimate of higher order contributions. The expansion factor then becomes  $\frac{Q}{\Lambda_\chi} \approx 0.35$ .

With this choice, we can estimate the 3NF contribution to the binding energy. Since it is of order  $Q^3$ , we expect that the 3NF contributes 4 % of the potential energy to the binding energy. In Table 2, we present the expectation values of the NN and 3N interactions for four different calculations of the  $^4\text{He}$  binding energy. The  $\text{N}^2\text{LO}$  calculations coincide with the ones we will use for the evaluation of the 4NF. Additionally, we show results for  $\text{N}^3\text{LO}$  calculations based on the chiral interactions of Ref. [20]. Note that also here the 3NF is only up to order  $\text{N}^2\text{LO}$ .

**Table 2.** BE's  $E$  and expectation values of the NN ( $\langle V_{NN} \rangle$ ) and 3N ( $\langle V_{3NF} \rangle$ ) interactions for  $^4\text{He}$ . All energies and the cutoffs are given in MeV. "DR" indicates that loops for the chiral interaction of Ref. [20] are regularized using dimensional regularization. The experimental BE is -28.30 MeV.

interaction	$\Lambda / \tilde{\Lambda}$	$E$	$\langle V_{NN} \rangle$	$\langle V_{3NF} \rangle$	$\langle V_{3NF} \rangle / \langle V_{NN} \rangle$
N <sup>2</sup> LO	450 / 700	-27.65	-84.56	-1.11	1.3 %
N <sup>2</sup> LO	600 / 700	-28.57	-93.73	-6.83	7.3 %
N <sup>3</sup> LO-3NF-A	500 / DR	-28.27	-99.45	-4.06	4.1 %
N <sup>3</sup> LO-3NF-B	500 / DR	-28.24	-98.92	-7.10	7.2 %



**Fig. 7.** 4NF contribution to the binding energy of  $^4\text{He}$ . Ten independent MC results are shown for the class IV contribution based on the LO wave function with  $\Lambda = 7 \text{ fm}^{-1}$ . Error bars are estimates for the single run standard deviations. The line is the average of all ten runs. The band indicates the standard deviation of the combined runs.

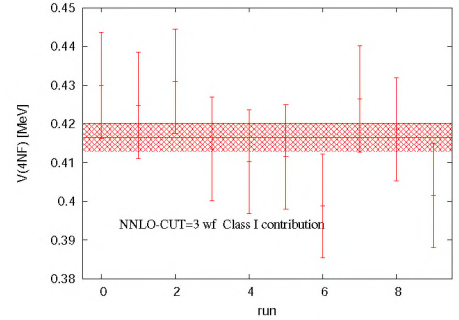
We find that the expectation values of the 3NF is indeed of the order of 4 %. This seems to back our very conservative choice for  $\Lambda_\chi$ . We note that the inclusion of  $\Delta$ 's into the effective theory shifts part of the 3NF to NLO. Here, we strictly stick to  $\Delta$ -less ChPT and estimate the higher order contributions based on  $\Lambda_\chi = 400 \text{ MeV}$ .

The 4NF is order  $Q^4$ . Based on the expectation values of NN potential given in the table, we can estimate that the 4NF contributes approximately 1 MeV to the binding energy. Such a contribution is not negligible in nuclear structure calculations. This estimate is also in line with the observation that an accurate description of NN data requires potentials up to order N<sup>3</sup>LO. It is therefore necessary to make an explicit calculation to get a more reliable estimate of its contribution.

## 5 Numerical technique

In this section, we want to introduce briefly the numerical method used for the evaluation of the pertinent integrals. Since we base our estimate of the 4NF contribution on first order perturbation theory, we need to calculate the expectation value of the 4NF with respect to  $^4\text{He}$  wave functions. This leads to integrals of the form

$$\langle V_4 \rangle = \sum_{\alpha\alpha'} \int d^3 p_{12} d^3 p_3 d^3 q_4 d^3 p'_{12} d^3 p'_3 d^3 q'_4 \langle \Psi | \mathbf{p}_{12} \mathbf{p}_3 \mathbf{q}_4 \alpha \rangle \langle \dots | V_4 | \dots \rangle \langle \mathbf{p}_{12}' \mathbf{p}_3' \mathbf{q}_4' \alpha' | \Psi \rangle$$



**Fig. 8.** Same as Fig. 7 for a calculation of the class I contribution for the NNLO wave function with  $\Lambda = 550 \text{ MeV}$  and  $\tilde{\Lambda} = 600 \text{ MeV}$ .

$$= \sum_{\alpha\alpha'} \int d^3 p_{12} d^3 p_3 d^3 q_4 d^3 p'_{12} d^3 p'_3 d^3 q'_4 w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4) \frac{\langle \Psi | \mathbf{p}_{12} \mathbf{p}_3 \mathbf{q}_4 \alpha \rangle \langle \dots | V_4 | \dots \rangle \langle \mathbf{p}_{12}' \mathbf{p}_3' \mathbf{q}_4' \alpha' | \Psi \rangle}{w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4)} \quad (1)$$

Here,  $\mathbf{p}_{12} \mathbf{p}_3 \mathbf{q}_4$  ( $\mathbf{p}_{12}' \mathbf{p}_3' \mathbf{q}_4'$ ) are incoming and outgoing Jacobi momenta in the 4N system. The 4NF matrix element is  $\langle \dots | V_4 | \dots \rangle$  and depends on these momenta and the incoming and outgoing spin/isospin channels  $\alpha$  and  $\alpha'$  (labeling all possible combinations of spin/isospin projections of the four nucleons). The  $^4\text{He}$  wave functions  $\langle \mathbf{p}_{12}' \mathbf{p}_3' \mathbf{q}_4' \alpha' | \Psi \rangle$  are also given in terms of the Jacobi momenta and  $\alpha$  and  $w$  is a weight function to be discussed below.

We have not performed a partial wave decomposition. Therefore, the dimensionality of the integral is much to high to be calculated with standard techniques. A Monte Carlo (MC) scheme is much better suited for this purpose. We found that an importance sampling similar to the Metropolis algorithm [41] is required to keep the computational needs small and increase the accuracy.

Usually such an importance sampling is guided by the square of the wave function. In configuration space, this quantity is perfectly suited as a weight function since it is then automatically normalized to one at least as long as the operators are local. For momentum space, the structure is more complicated, since the integrals require weight functions with higher dimensionality as in configuration space. This implies that a simple square of the wave function is not useful for the importance sampling anymore. This problem could be solved by performing part of the

integrals using standard methods as has been successfully done in [42]. We found this approach less practical in our case, since the three- and four-nucleon operators would require to perform high dimensional integrations using standard integration methods.

Our solution was to give up weight functions based on the wave functions of the system, but choose a rational ansatz instead. The parameters of the ansatz were then adjusted so that the standard deviation in test cases was minimized. In this way, we were able to improve the accuracy sufficiently. At the same time, the weight function could be analytically normalized to one so that the calculations became feasible.

E.g. we choose for the importance sampling for integrals of the form of Eq. (1) a weight function depending on the six integration variables  $\mathbf{p}_i = \mathbf{p}_{12}, \mathbf{p}_3, \mathbf{q}_4, \mathbf{p}'_{12}, \mathbf{p}'_3$  and  $\mathbf{q}'_4$

$$w(\mathbf{p}'_{12}, \mathbf{p}'_3, \mathbf{q}'_4, \mathbf{p}_{12}, \mathbf{p}_3, \mathbf{q}_4) \equiv w(p'_{12}, p'_3, q'_4, p_{12}, p_3, q_4) \\ = \prod_i \frac{(r-3)(r-2)(r-1)}{8\pi} \frac{C_{p_i}^{(r-3)}}{(p_i + C_{p_i})^r} \quad (2)$$

For simplicity, the ansatz only depends on the magnitude of the momenta. With the parameters  $C_{p_i}$  and  $r$  the shape of the weight functions can be influenced. The ansatz guarantees (for large enough  $r$ ) that the weight function is normalized to one.

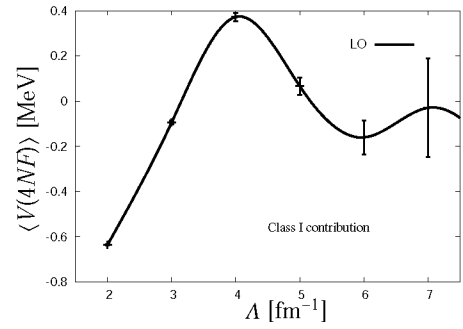
In practice, we used a *Mathematica* script to generate the numerical expressions of the potential matrix elements for each  $\alpha/\alpha'$  reliably. The resulting code lines could be directly included in a FORTRAN code evaluating the high dimensional integrals given above.

In order to check the statistical character of our MC results, we performed for each quantity 10 independent calculations. Figs. 7 and 8 summarize the results of two representative sets of calculations. The error bars are standard deviations of the single runs. The line is the average of all the ten runs and the shaded region indicates the standard deviation of the average. It is reassuring that the approximately 2/3 of the single runs overlap with the average in both cases within one standard deviation. This is a non-trivial confirmation of the statistics of the runs.

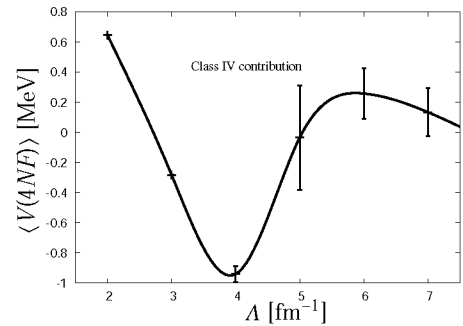
With this technique, we were able to estimate the 4NF contribution for the different interactions reliably. We are now in the position to discuss the results in the next section.

## 6 Results

We start the discussion of the results based on LO wave functions for which we were able to investigate a large range of cutoffs. Some exemplary results are shown in Figs. 9 and 10. The actual contribution of the two shown parts of the 4NF are strongly cutoff dependent. This is not surprising and reflects the fact that the potential is not observable. Even for the large cutoff the 4NF contribution is stable and remains below or around 1 MeV. This is not only true for these two examples, but also for all the other classes of diagrams.



**Fig. 9.** Cutoff dependence of the class I contribution to the 4NF for the LO wave functions. Error bars are the statistical errors of the MC evaluation.



**Fig. 10.** Cutoff dependence of the class IV contribution to the 4NF for the LO wave functions. Error bars are the statistical errors of the MC evaluation.

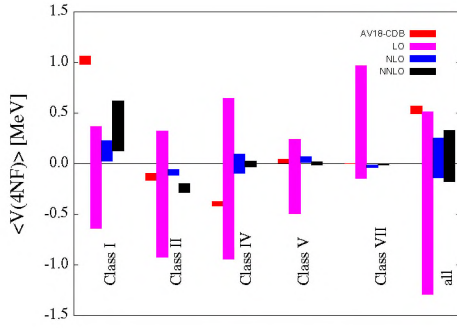
For the larger cutoffs, it is not necessarily true that the 4NF is perturbative. For 3NF's, we know that first order perturbation theory is insufficient for some higher cutoffs [43] or for some phenomenological models [44]. Therefore, the large cutoff results have to be taken with some care, although we have no indication that perturbation theory is not appropriate for these estimates.

Surprisingly, we find a rather large contribution from class IV. This class depends on  $C_T$ , which generally is smaller than natural due to Wigner symmetry. But our results for the expectation values in LO for class IV is not unnaturally small. In fact, as can be seen in the figures, it is larger than class I contributions for most cutoffs. At this point, we do not fully understand this enhancement of the LO results.

The complete final results are depicted in Fig. 11. We show results for the different classes separately. The bars indicate the range of the results for different cutoffs (or for AV18 and CD-Bonn for the phenomenological models). For the chiral interactions, the parameters of the 4NF are completely fixed by the NN interaction. For the phenomenological ones, the strength of the contact pieces was fixed arbitrarily to  $C_T = 10 \text{ GeV}^{-2}$ . This choice is below the naturalness estimate and is meant to take Wigner symmetry into account. For completeness, we also show the sum of all contributions.

The phenomenological and LO estimates tend to be larger than the ones for NLO and N<sup>2</sup>LO interactions. In both cases, we observe that class IV contributions are larger





**Fig. 11.** Expectation values of the 4NF for various chiral and phenomenological interactions. Contributions of different classes are shown separately. The width of the bars indicates the dependence on the cutoff for the chiral interactions and the band spanned by AV18 and CD-Bonn for the phenomenological interactions.

than expected by the size of  $C_T$ . We also note that the CD-Bonn and AV18 results are close to each other, although the LO results are strongly cutoff dependent. This is an unusually behavior, since generally cutoff dependence for LO results shows up as a strong model dependence for phenomenological calculations. It has to be seen in future, whether this dependence can be traced back to the cutoff dependence of the binding energy.

NLO and N<sup>2</sup>LO results are more interesting since for these the NN interactions are strictly consistent with the 4NF and the binding energy is already described reasonably. The results for these interactions are smaller. As expected, the class IV, V and VII contributions are suppressed because of Wigner symmetry. Individually, the class I and class II contributions are of the order of 500 keV. The sum of both is smaller (around 300 keV) since both parts cancel each other partly. We also note that for some cutoffs the 4NF acts attractively or repulsively. We again stress that the potential is not observable. Therefore, we cannot expect results to be independent of the cutoff.

## 7 Conclusions and outlook

In summary, we have studied the 4NF contribution to the binding energy of  ${}^4\text{He}$  in the framework of chiral perturbation theory. To this aim, we made use of a MC technique in momentum space that enabled us to calculate the high dimensional integrals required for the evaluation of the expectation values. The scheme allows one to generate the most complex parts of the code using *Mathematica*. This way, the expressions can be reliably transferred into our FORTRAN codes.

By now it is clear that 3NF's give important contributions to the binding energies of nuclei. Based on the power counting, 4NF contributions might still be significant. We found by explicit calculation that the 4NF contribution is somewhat smaller than the power counting estimate of 1 MeV at least when the higher order chiral interactions are used. The individual contributions of class IV to VII are suppressed due to Wigner symmetry, so that only class I and II contributions are non-negligible. For

${}^4\text{He}$  these two classes cancel each other in parts so that the complete contribution for this case is below or up to approximately 300 keV in magnitude.

Although this would probably be considered as a negligible contribution, some care has to be taken before final conclusion on 4NF's can be made. Firstly, the phenomenological interactions tend to lead to larger 4NF. But most importantly, the 4NF contribution could be larger for nuclei with a different spin/isospin structure than  ${}^4\text{He}$ . In this case the class I and II contributions might add constructively implying a visible contribution of 4NF's. This has to be studied in more detail in future.

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