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Relativistic Effects in Neutron–Deuteron Elastic Scattering and Breakup

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Abstract We solved the Faddeev equation in a Poincaré invariant model of the three-nucleon system. Two-body interactions are generated so that when they are added to the two-nucleon invariant mass operator (rest energy) the two-nucleon S matrix is identical to the experimental S matrix modeled with a given nucleon–nucleon interaction. Cluster properties of the three-nucleon S-matrix determine how these two-nucleon interactions are embedded in the three-nucleon mass operator. Differences in the predictions of the relativistic and corresponding non-relativistic models for elastic and breakup processes are investigated. Of special interest are effects of relativity on the elastic scattering angular distribution and total cross sections, the lowering of the A_y maximum in elastic nucleon-deuteron (Nd) scattering below ≈25 MeV caused by the Wigner spin rotations and the significant changes of the breakup cross sections in certain regions of the phase-space.

1 Introduction

The study of elastic and breakup nucleon-deuteron (Nd) processes reveals cases where the non-relativistic description based on nucleon–nucleon (NN) interactions only is insufficient to explain the data. These discrepancies generally increase with energy. The elastic Nd angular distribution in the region of its minimum and at backward angles is the best studied example [1] (see Fig.1). Another one is the total cross section for neutron–deuteron (nd) scattering [2,3]. Only in some cases the inclusion of certain types of three-nucleon (3N) forces lead to an improvement. A relativistic treatment of the dynamics implies a different off-shell treatment of the NN interactions [4], leading to the possibility of additional effects beyond standard 3N forces. We refer to [5–8] for a detailed presentation and focus here on effects of relativity on the Nd elastic scattering angular distribution and total cross sections, the A_y puzzle and pronounced relativistic effects in Nd breakup.

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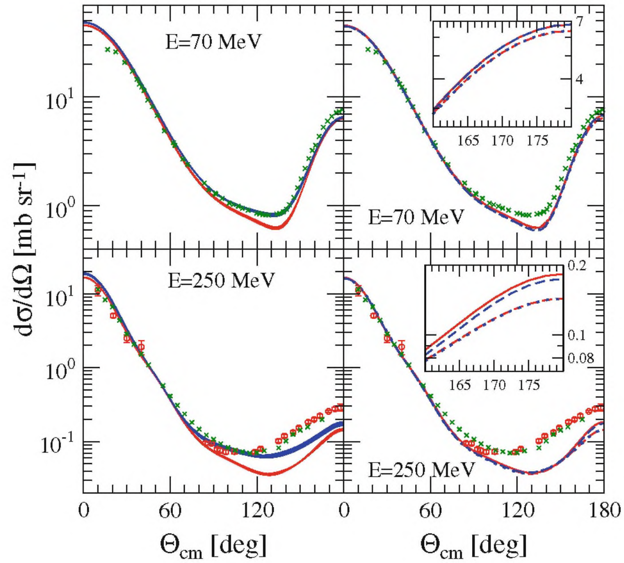

Fig. 1 (Color online) The Nd elastic scattering cross sections at 70 and 250 MeV lab. energy of the incoming nucleon. In the *left column* predictions based on AV18 [13], CD Bonn [14], NijmI and Nijm II [15] NN potentials alone and on their combinations with the TM99 3NF [16] are shown by the *light shaded* (*red*) and *dark shaded* (*blue*) band, respectively. In the *right column* the relativistic predictions based on the boosted CD Bonn ((*red*) *solid line*) and AV18 ((*blue*) *dashed line*) potentials are compared with the nonrelativistic CD Bonn ((*red*) *dashed line*) and AV18 ((*blue*) *dotted line*) predictions. The 70 MeV pd data (x-es) are from [17]. The 250 nd (x-es) and pd (*open circles*) data are from [18] and [19], respectively

2 Relativistic Faddeev Equation

The 3N Faddeev equation is set up for a breakup operator and solved in momentum space and partial wave projected. In the relativistic case the equations have the same operator form as the non-relativistic equations. In the relativistic case Jacobi momenta are constructed using Lorentz boosts instead of Galilean boosts, the resolvents involve relativistic kinetic energies, the two-body interactions in the three-body problem appear inside of square roots in a manner dictated by S-matrix cluster properties, and the permutation operators include Wigner rotations [9] which are evaluated using the Balian–Brézin method [10].

3 Results

In Fig. 1 we present relativistic effects for the nd elastic scattering cross sections at two energies. Only at the largest angles deviations from the nonrelativistic results are discernible and their magnitude grows with energy. Relativity increases slightly the nonrelativistic cross sections at backward angles but the effect is far too small to explain the large gap between data and theory even when standard three-nucleon forces are included.

In Fig. 2 we show relativistic effects for A_y at two low energies. Below ≈25 MeV the non-linear embedding of the two-body interaction in the three-body mass operator lowers the maximum of A_y by ≈2% while the inclusion of Wigner spin rotations increase that lowering effect up to ≈10%. Above ≈25 MeV relativistic effects for A_y are negligible.

For the breakup cross section large relativistic effects are localized in specific regions of phase-space. They lead to a characteristic pattern of relativistic versus non relativistic cross section changes. Namely, at fixed detector position of the first outgoing nucleon changing the angle of the second outgoing nucleon lead to configurations in which relativity increases, makes no effect, or decreases the nonrelativistic cross section. That is exemplified in Fig. 3. At $E_N^{\mathrm{lab}} = 200$ MeV those changes can be up to $\approx \pm 60\%$. At that energy it seems that relativity improves the description of some data (see Fig. 3).

In spite of large relativistic effects in some exclusive breakup configurations the effects in total cross sections are small. In Table 1 we compare nonrelativistic and relativistic total cross sections for nd scattering as well as the corresponding total cross sections for elastic nd scattering and breakup reaction. Relativity lowers slightly the total cross sections.

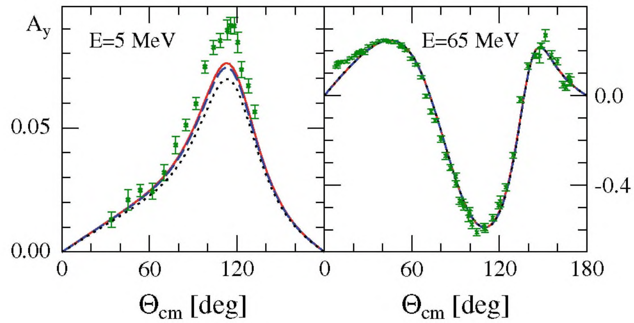

Fig. 2 (Color online) The nucleon analyzing power A_y for nd elastic scattering at 5 and 65 MeV lab. energy of the incoming neutron. The *solid* (*red*) *line* is the nonrelativistic CD Bonn potential prediction. The *dashed* (*blue*) and *dotted* (*black*) *lines* are CD Bonn based relativistic predictions without and with Wigner spin rotations, respectively. The 5 MeV nd data (x-es) are from [20] and 65 MeV pd data (x-es) are from [21]

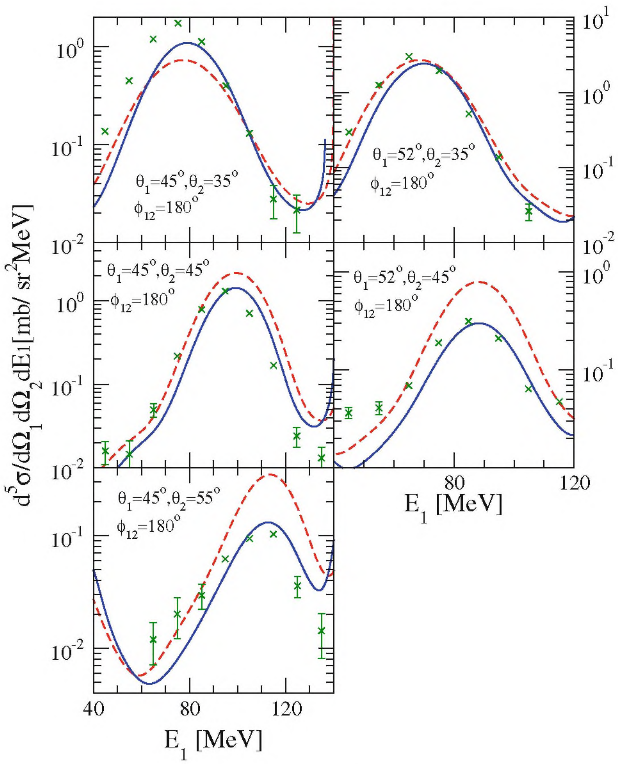

Fig. 3 (Color online) The fivefold cross section $d^5\sigma/d\Omega_1 d\Omega_2 dE_1^{\rm lab}$ for the breakup reaction d(n,np)n at $E_{\rm lab} = 200$ MeV and fixed angles of outgoing nucleons 1 and 2 as indicated in the figures. The *dashed* (*red*) *line* is the nonrelativistic CD Bonn potential prediction and the *solid* (*blue*) *line* is the corresponding relativistic result. The d(p,pn)p data (x-es) are from [22]

4 Summary

An exactly Poincaré invariant formulation of three-nucleon scattering using realistic interactions leads to significant changes of the breakup cross section at higher energies and in certain regions of phase space [6,7]. For the elastic scattering cross sections the small changes are restricted to backward angles [5]. Also relativistic effects are small for total cross sections. For the low energy analyzing power A_y we found large relativistic effects of similar magnitude as in [11] but in opposite direction and that increases the discrepancy to the data. Therefore we expect that 3NF's in all their complexity [12] have to be taken into account.

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Table 1 The AV18 and CD Bonn potential relativistic (r) and nonrelativistic (nr) total cross sections

$E_{\rm lab}$ (MeV)	$\sigma_{\rm tot}^{\rm r}$ (mb)	$\sigma_{\rm tot}^{\rm nr}$ (mb)	$\sigma_{\rm el}^{\rm r}$ (mb)	$\sigma_{\rm el}^{\rm nr}$ (mb)	$\sigma_{\rm br}^{r}$ (mb)	$\sigma_{\rm br}^{\rm nr}$(mb)
AV18						
13.0	855.44	856.84	692.10	686.70	163.42	164.74
26.0	455.27	457.05	303.57	304.14	151.70	152.91
70.0	145.83	146.95	61.57	61.86	84.26	85.09
97.5	100.66	101.63	34.83	35.02	65.83	66.61
135.0	74.73	75.65	20.82	20.94	53.91	54.70
200.0	58.60	59.62	12.20	12.28	46.40	47.34
250.0	53.54	54.74	9.50	9.58	44.05	45.16
CD Bonn						
13.0	853.16	854.25	687.14	686.70	166.02	167.55
26.0	457.50	459.15	304.01	304.50	153.48	154.65
70.0	147.89	148.96	62.50	62.75	85.39	86.21
97.5	102.21	103.16	35.39	35.56	66.83	67.61
135.0	75.77	76.70	21.05	21.17	54.72	55.53
200.0	59.11	60.13	12.24	12.30	46.87	47.83
250.0	53.85	54.98	9.52	9.57	44.33	45.41

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