Few-Body Syst (2011) 50: 279–281
DOI 10.1007/s00601-010-0204-7

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Recent Developments of a Three-dimensional Description of the NN System

Received: 29 October 2010 / Accepted: 18 December 2010 / Published online: 7 January 2011

Abstract A recently developed three-dimensional formulation of nucleon–nucleon (NN) scattering is briefly presented. Here the NN t-matrix is represented by six spin-momentum operators accompanied by six scalar functions of momentum vectors. A numerical example for the NN scattering cross section is given.

Traditionally numerical calculations for two- or three-nucleon systems are based on the partial wave decomposition (PWD) of states taking advantage of the rotational invariance of the interactions involved. This is an established and successful procedure for describing processes at moderate projectile energies. However, if one wishes to move to a higher energy regime, the number of partial waves needed to obtain a converged result proliferates and a scheme that avoids a PWD becomes more appealing.

Recently we proposed a three-dimensional (3D) formulation of the momentum space Faddeev equations for 3N bound states [1] and 3N scattering [2] taking advantage of the fact that traces over spin-momentum operators can be analytically evaluated, leaving the Faddeev equations as a finite set of coupled equations for

Presented at the 21st European Conference on Few-Body Problems in Physics, Salamanca, Spain, 30 August–3 September 2010.

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scalar functions which only depend on vector momenta. This very idea was used to calculate NN scattering in Ref. [3].

In this formulation we use that for a fixed two-nucleon isospin t, any NN interaction, and thus the NN t-matrix, can be expanded as a sum of six independent scalar spin-momentum operators w_j accompanied by scalar functions v_j^t, which only depend on the initial and final momenta $\mathbf{p}$ and $\mathbf{p}'$. For fixed t the potential is given as

$$V^t(\mathbf{p}', \mathbf{p}) \equiv \sum_{j=1}^{6} v_j^t(\mathbf{p}', \mathbf{p})\; w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}). \tag{1}$$

We use the following set of operators:

$$\begin{aligned}
w_1(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= 1 \\
w_2(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
w_3(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= i\; (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{p} \times \mathbf{p}') \\
w_4(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathbf{p} \times \mathbf{p}')\; \boldsymbol{\sigma}_2 \cdot (\mathbf{p} \times \mathbf{p}') \\
w_5(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathbf{p}' + \mathbf{p})\; \boldsymbol{\sigma}_2 \cdot (\mathbf{p}' + \mathbf{p}) \\
w_6(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathbf{p}' - \mathbf{p})\; \boldsymbol{\sigma}_2 \cdot (\mathbf{p}' - \mathbf{p}).
\end{aligned} \tag{2}$$

Substituting the expansion of Eq. (1) for V as well as t into the Lippmann–Schwinger (LS) equation leads to a set of six coupled equations for the scalar functions $t_j^t(\mathbf{p}', \mathbf{p})$, which only depend on the magnitudes of the vectors $\mathbf{p}$ and $\mathbf{p}'$ and the angle $\hat{\mathbf{p}}' \cdot \hat{\mathbf{p}}$ between them. Then the LS equation reads

$$\begin{aligned}
\sum_j A_{kj}(\mathbf{p}', \mathbf{p}) t_j^{tm_t}(\mathbf{p}', \mathbf{p}) &= \sum_j A_{kj}(\mathbf{p}', \mathbf{p}) v_j^{tm_t}(\mathbf{p}', \mathbf{p}) \\
&+ \int d^3p'' \sum_{jj'} v_j^{tm_t}(\mathbf{p}', \mathbf{p}'') G_0(p'')\; t_{j'}^{tm_t}(\mathbf{p}'', \mathbf{p})\; B_{kjj'}(\mathbf{p}', \mathbf{p}'', \mathbf{p}),
\end{aligned} \tag{3}$$

where G_0 is the free two-nucleon propagator. The scalar functions $A_{kj}(\mathbf{p}', \mathbf{p})$ and $B_{kjj'}(\mathbf{p}', \mathbf{p})$ are defined as

$$A_{kj}(\mathbf{p}', \mathbf{p}) \equiv \mathrm{Tr}\left(w_k(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p})\; w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p})\right) \tag{4}$$

$$B_{kjj'}(\mathbf{p}', \mathbf{p}'', \mathbf{p}) \equiv \mathrm{Tr}\left(w_k(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p})\; w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}'')\; w_{j'}(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}'', \mathbf{p})\right) \tag{5}$$

All traces over the spin-momentum operators are evaluated analytically, resulting in functions which only depend on the vectors $\mathbf{p}$, $\mathbf{p}'$, and $\mathbf{p}''$ and angles between them. The coefficient functions A_{kj} and $B_{kjj'}$ only need to be calculated once. Explicit expressions for them are given in [3].

After solving Eq. (3) for the t-matrix, NN observables are calculated with standard methods, as e.g. given in Ref. [4]. For all the details of the formulation, the properties of on-shell NN t-matrix and the numerical realization we refer to Ref. [3].

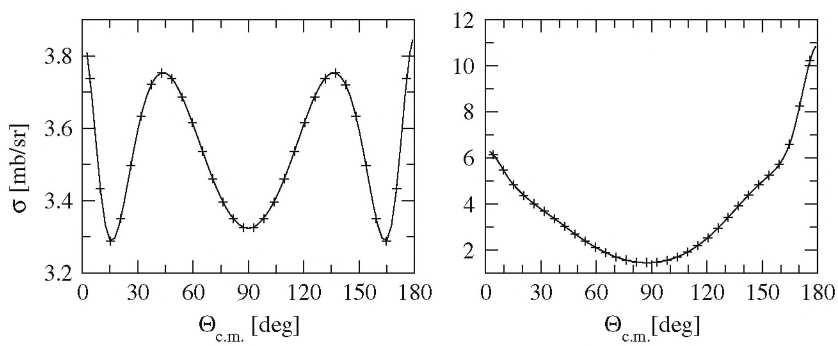

Fig. 1 The differential cross section for neutron–neutron (*right*) and neutron–proton (*left*) scattering at the projectile laboratory kinetic energy $\mathrm{E}_{lab} = 300$ MeV as a function of the center of mass angle θ. *Crosses* represent the calculations based on the 3D approach and the *solid lines* represent results from the standard PWD. Both calculations are based on the Bonn-B potential

When considering the deuteron which carries isospin $t = 0$ and total spin $s = 1$, we start from an operator equation [1] and again use the expansion of Eq. (1). This leads to a set of two coupled equations for the s- and d-wave components with coefficients $A^d_{kk'}(p)$ and $B^d_{kjk''}(\mathbf{p}, \mathbf{p}')$, which again are scalar functions of vector momenta. As for NN scattering, those can be calculated once for all and are explicitly given in [3].

We achieved excellent agreement between the 3D and the partial wave based calculations for NN scattering observables, as is exemplified in Fig. 1 for the Bonn-B [5] NN potential by the cross section at E_{lab}=300 MeV. The operator form of the Bonn-B potential and its parameters are given in [3].

Summarizing, we described a new approach to treat the NN system directly in three dimensions using momentum vectors. This approach is based on an expansion of the NN potential in a set of six linear independent momentum-spin operators multiplied by scalar functions of the momentum vectors. The NN scattering observables obtained in this scheme and, not shown here, deuteron properties are in a perfect agreement with calculations based on a conventional PWD. This confirms the correctness of the new approach and can serve as starting point for a 3D description of three and more nucleon systems.

Acknowledgments This work was supported by the Polish Ministry of Science and Higher Education under Grants No. N N202 104536 and No. N N202 077435 and in part under the auspices of the U. S. Department of Energy, Office of Nuclear Physics under contract No. DE-FG02-93ER40756 with Ohio University. It was also partially supported by the Helmholtz Association through funds provided to the virtual institute “Spin and strong QCD”(VH-VI-231). The numerical calculations were partly performed on the supercomputer cluster of the JSC, Jülich, Germany.

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