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Threshold resummation for hadronic collisions with three particles in the final state

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In the following we summarize our results on resummation of threshold corrections for the $2 \to 3$ class of hadronic production processes in the Mellin moment space formalism and apply our results to the associated Higgs boson production process $pp \to t\bar{t}H$ at the LHC.

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1. Introduction

One of the main tasks of the current LHC run is to establish the properties of the Higgs boson discovered at the LHC in 2012 [1, 2]. The production process in association with top quarks, $pp \to t\bar{t}H$, provides a direct way to probe the strength of the top Yukawa coupling without making any assumptions regarding its nature. This necessitates an improvement of the theoretical accuracy with which theoretical predictions for $pp \to t\bar{t}H$ are known. A great amount of progress has been achieved in the recent years in this field. Although the next-to-leading-order (NLO) QCD, i.e. $\mathcal{O}(\alpha_s^3\alpha)$ predictions are already known for some time [3, 4], they have been newly recalculated and matched to parton showers in [5, 6, 7, 8]. As of late, the mixed QCD-weak corrections [9] and QCD-EW corrections [10, 11] of $\mathcal{O}(\alpha_s^2\alpha^2)$, as well as the NLO QCD corrections to the hadronic $t\bar{t}H$ production with top and antitop quarks decaying into bottom quarks and leptons [12] are also available. However, calculations of the next-to-next-to-leading-order QCD corrections are currently technically out of reach. It is nevertheless interesting to ask the question what is the size and the effect of certain classes of QCD corrections of higher than NLO accuracy. In this work we focus on taking into account, to all orders in perturbation theory, contributions from soft gluon emission arising in the threshold limit.

The traditional (Mellin-space) resummation formalism which is often applied in this type of calculations has been very well developed and copiously employed for description of the $2 \to 2$ type processes at the Born level. The universality of resummation concepts warrants their applications to scattering processes with many partons in the final state, as shown in a general analytical treatment developed for arbitrary number of partons [13, 14]. In particular, using a concept of individual weights for each of the functions describing different type of dynamics, be it hard, soft/collinear or soft, the factorization of the cross sections into these functions can be shown [15]. At the level of a specific process, adding one more particle or a jet in the final state requires accounting for more complicated kinematics and a possible change in the colour structure of the underlying hard scattering. In the general framework the former will manifest itself in the appearence of new type of weights, stricly related to the definition of a considered observable, while the latter influences the soft and hard functions. More specifically, for processes with more than three partons involved at the Born level, the non-trivial colour flow influences the contributions from wide-angle soft gluon emissions which have to be included at the next-to-leading-logarithmic (NLL) accuracy. The evolution of the colour exchange at NLL is governed by the one-loop soft anomalous dimension which then needs to be calculated.

In the following we discuss these modifications for a generic $ij \to klB$ process, where i, j denote massless coloured partons, k, l are two massive coloured particles and B is a massive colour-singlet particle, considered in the limit of absolute threshold with the corresponding weight given by $\beta^2 = 1 - (m_k + m_l + m_B)^2/\hat{s}$. Subsequently we apply the results to the case of the associated Higgs boson production with top quarks [16], where in the absolute threshold limit the cross section receives enhancements in the form of logarithmic corrections in β. The quantity β measures the distance from absolute production threshold and can be related to the maximal velocity of the $t\bar{t}$ system. An additional improvement of the calculation at the NLL accuracy is achieved by including the $\mathcal{O}(\alpha_s)$ non-logarithmic threshold corrections originating from hard off-shell dynamics.

2. Resummation at production threshold

At the partonic level, the Mellin moments for the process $ij \to klB$ are given by

$$\hat{\sigma}_{ij\to klB,N}(m_k, m_l, m_B, \mu_F^2, \mu_R^2) = \int_0^1 d\hat{\rho}\, \hat{\rho}^{N-1} \hat{\sigma}_{ij\to klB}(\hat{\rho}, m_k, m_l, m_B, \mu_F^2, \mu_R^2) \tag{2.1}$$

with $\hat{\rho} = 1 - \beta^2 = M^2/\hat{s}$, $M = m_l + m_k + m_B$.

At LO, the $t\bar{t}H$ production receives contributions from the $q\bar{q}$ and gg channels. We analyze the colour structure of the underlying processes in the s-channel color bases, $\{c_I^q\}$ and $\{c_I^g\}$, with $c_{\mathbf{1}}^q = \delta^{\alpha_i\alpha_j}\delta^{\alpha_k\alpha_l}$, $c_{\mathbf{8}}^q = T^a_{\alpha_i\alpha_j}T^a_{\alpha_k\alpha_l}$, $c_{\mathbf{1}}^g = \delta^{a_i a_j}\delta^{\alpha_k\alpha_l}$, $c_{\mathbf{8S}}^g = T^b_{\alpha_l\alpha_k} d^{ba_ia_j}$, $c_{\mathbf{8A}}^g = iT^b_{\alpha_l\alpha_k} f^{ba_ia_j}$. In this basis the soft anomalous dimension matrix becomes diagonal in the production threshold limit [17] and the the NLL resummed cross section in the N-space has the form [17, 18]

$$\hat{\sigma}^{(\mathrm{res})}_{ij\to klB,N} = \sum_I \hat{\sigma}^{(0)}_{ij\to klB,I,N}\, C_{ij\to klB,I}\, \Delta^i_{N+1}\Delta^j_{N+1}\Delta^{(\mathrm{int})}_{ij\to klB,I,N+1}, \tag{2.2}$$

where we suppress explicit dependence on the scales. The index I in Eq. (2.2) distinguishes between contributions from different colour channels. The colour-channel-dependent contributions to the LO partonic cross sections in Mellin-moment space are denoted by $\hat{\sigma}^{(0)}_{ij\to klB,I,N}$. The radiative factors Δ^i_N describe the effect of the soft gluon radiation collinear to the initial state partons and are universal, see e.g. [18] . Large-angle soft gluon emission is accounted for by the factors $\Delta^{(\mathrm{int})}_{ij\to klB,I,N}$ which are directly related to the soft gluon anomalous dimension calculated in [16]. As indicated by the lower indices, the wide-angle soft emission depends on the partonic process under consideration and the colour configuration of the participating particles. In the limit of absolute threshold production $\beta \to 0$, the factors $\Delta^{(\mathrm{int})}_{ij\to klB,I,N}$ coincide with the corresponding factors for a $2 \to 2$ process $ij \to kl$ [16]. In our calculations we consider all perturbative functions governing the radiative factors up to the terms needed to obtain NLL accuracy in the resummed expressions.

The coefficients $C_{ij\to klB,I} = 1 + \frac{\alpha_s}{\pi} C^{(1)}_{ij\to klB,I} + \ldots$ contain all non-logarithmic contributions to the NLO cross section taken in the threshold limit. More specifically, these consist of Coulomb corrections, N-independent hard contributions from virtual corrections and N-independent non-logarithmic contributions from soft emissions. Although formally the coefficients $C_{ij\to klB,I}$ begin to contribute at NNLL accuracy, in our numerical studies of the $pp \to t\bar{t}H$ process we consider both the case of $C_{ij\to klB,I} = 1$, i.e. with the first-order corrections to the coefficients neglected, as well as the case with these corrections included. In the latter case we treat the Coulomb corrections and the hard contributions additively, i.e. $C^{(1)}_{ij\to klB,I} = C^{(1,\mathrm{hard})}_{ij\to klB,I} + C^{(1,\mathrm{Coul})}_{ij\to klB,I}$. For k,l denoting massive quarks the Coulomb corrections are $C^{(1,\mathrm{Coul})}_{ij\to klB,\mathbf{1}} = C_{\mathrm{F}}\pi^2/(2\beta_{kl})$ and $C^{(1,\mathrm{Coul})}_{ij\to klB,\mathbf{8}} = (C_{\mathrm{F}} - C_{\mathrm{A}}/2)\pi^2/(2\beta_{kl})$ with $\beta_{kl} = \sqrt{1 - 4m_t^2/\hat{s}_{kl}}$ and $\hat{s}_{kl} = (p_t + p_{\bar{t}})^2$. As the N-independent non-logarithmic contributions from soft emission are accounted for using the techniques developed for the $2 \to 2$ case [19, 20], the problem of calculating the $C^{(1)}_{ij\to t\bar{t}H,I}$ coefficients reduces to calculation of virtual corrections to the process. We extract them numerically using the publicly available POWHEG implementation of the $t\bar{t}H$ process [8], based on the calculations developed in [4]. The results are then cross-checked using the standalone MadLoop implementation in aMC@NLO [5]. Since the $q\bar{q}$ channel receives only colour-octet contributions, the extracted value contributing to $C^{(1,\mathrm{hard})}_{q\bar{q}\to t\bar{t}H,\mathbf{8}}$ is exact. In the gg channel, however, both the singlet and octet production modes contribute. We extract the value which contributes to the coefficient $\bar{C}^{(1,\text{hard})}_{gg\to t\bar{t}H}$ averaged over colour channels and use the same value to further calculate $C_{gg\to t\bar{t}H,\mathbf{1}}$ and $C_{gg\to t\bar{t}H,\mathbf{8}}$.

The resummation-improved NLO+NLL cross sections for the $pp \to t\bar{t}H$ process are then obtained through matching the NLL resummed expressions with the full NLO cross sections

$$\sigma^{(\text{NLO+NLL})}_{h_1h_2\to klB}(\rho,\mu_F^2,\mu_R^2) = \sigma^{(\text{NLO})}_{h_1h_2\to klB}(\rho,\mu_F^2,\mu_R^2) + \sigma^{(\text{res}-\text{exp})}_{h_1h_2\to klB}(\rho,\mu_F^2,\mu_R^2)$$

with

$$\sigma^{(\text{res}-\text{exp})}_{h_1h_2\to klB} = \sum_{i,j}\int_{\mathscr{C}}\frac{dN}{2\pi i}\,\rho^{-N} f^{(N+1)}_{i/h_1}(\mu_F^2)\, f^{(N+1)}_{j/h_2}(\mu_F^2) \times\left[\hat{\sigma}^{(\text{res})}_{ij\to klB,N}(\mu_F^2,\mu_R^2) - \hat{\sigma}^{(\text{res})}_{ij\to klB,N}(\mu_F^2,\mu_R^2)\,|_{(NLO)}\right], \quad (2.3)$$

where $\hat{\sigma}^{(\text{res})}_{ij\to klB,N}$ is given in Eq. (2.2) and $\hat{\sigma}^{(\text{res})}_{ij\to klB,N}\,|_{(NLO)}$ represents its perturbative expansion truncated at NLO. The moments of the parton distribution functions (pdf) $f_{i/h}(x,\mu_F^2)$ are defined in the standard way $f^{(N)}_{i/h}(\mu_F^2) \equiv \int_0^1 dx x^{N-1} f_{i/h}(x,\mu_F^2)$. The inverse Mellin transform (2.3) is evaluated numerically using a contour $\mathscr{C}$ in the complex-N space according to the "Minimal Prescription" method developed in Ref. [21].

3. Numerical predictions

The numerical results presented in this section are obtained with $m_t = 173$ GeV, $m_H = 125$ GeV and MMHT14 pdf sets [22]. We choose the central renormalization and factorization scales as $\mu_{F,0} = \mu_{R,0} = m_t + m_H/2$, in accordance with [23]. The NLO cross section is calculated using the aMC@NLO code [24].

In figure 1 we analyse the scale dependence of the resummed total cross section for $pp \to t\bar{t}H$ at $\sqrt{S} = 8$ and 14 TeV, varying simultaneously the factorization and renormalization scales, μ_F and μ_R. As demonstrated in Fig. 1, adding the soft gluon corrections stabilizes the dependence on $\mu = \mu_F = \mu_R$ of the NLO+NLL predictions with respect to NLO. The central values, calculated at $\mu = \mu_0 = m_t + m_H/2$, and the scale error at $\sqrt{S} = 8$ TeV changes from $132^{+3.9\%}_{-9.3\%}$ fb at NLO to $141^{+7.7\%}_{-4.6\%}$ fb at NLO+NLL (with $C^{(1)}_{ij\to t\bar{t}H,I}$ coefficients included) and correspondingly, from $641^{+0.8\%}_{-1.3\%}$ to $650^{+7.9\%}_{-5.7\%}$ fb at $\sqrt{S} = 14$ TeV. It is also clear from figure 1 that the coefficients $C^{(1)}_{ij\to t\bar{t}H}$ strongly impact the predictions, especially at higher scales. In fact, their effect is more important than the effect of the logarithmic corrections alone, in correspondence to the strong suppression $\sim \beta^4$ for the real emission in the $2\to 3$ process due to the massive three particle phase space. This observation also indicates the relevance of the contributions originating from the region away from the absolute threshold which need to be known in order to further improve theoretical predictions.

The effect of including NLL corrections is summarized in Table 1 for the LHC collision energy of 8, 13 and 14 TeV. Here we choose to estimate the theoretical uncertainty due to scale variation using the 7-point method, where the minimum and maximum values obtained with $(\mu_F/\mu_0, \mu_R/\mu_0) = (0.5,0.5),(0.5,1),(1,0.5),(1,1),(1,2),(2,1),(2,2)$ are considered. The NLO+NLL predictions show a significant reduction of the scale uncertainty, compared to NLO results. The reduction of the positive and negative scale errors amounts to around 20-30% of the NLO error for $\sqrt{S} = 13, 14$ TeV. This general reduction trend is not sustained for the positive error after including the $C^{(1)}_{ij\to t\bar{t}H,I}$

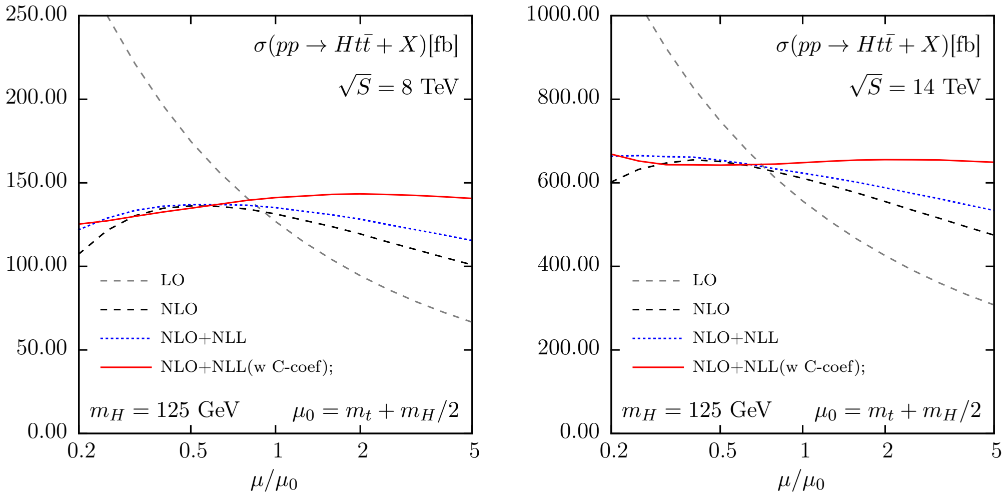

Figure 1: Scale dependence of the LO, NLO and NLO+NLL cross sections at $\sqrt{S} = 13$ and $\sqrt{S} = 14$ TeV LHC collision energy. The results are obtained while simultaneously varying μ_F and μ_R, $\mu = \mu_F = \mu_R$.

$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL		NLO+NLL with C		pdf error
		Value [fb]	K-factor	Value [fb]	K-factor	
8	$132^{+3.9\%}_{-9.3\%}$	$135^{+3.0\%}_{-5.9\%}$	1.03	$141^{+7.7\%}_{-4.6\%}$	1.07	$^{+3.0\%}_{-2.7\%}$
13	$506^{+5.9\%}_{-9.4\%}$	$516^{+4.6\%}_{-6.5\%}$	1.02	$537^{+8.2\%}_{-5.5\%}$	1.06	$^{+2.3\%}_{-2.3\%}$
14	$613^{+6.2\%}_{-9.4\%}$	$625^{+4.6\%}_{-6.7\%}$	1.02	$650^{+7.9\%}_{-5.7\%}$	1.06	$^{+2.3\%}_{-2.2\%}$

Table 1: NLO+NLL and NLO total cross sections for $pp \to t\bar{t}H$ at $\sqrt{s} = 8,\ 13$ and 14 GeV. The error ranges given together with the NLO and NLO+NLL results indicate the scale uncertainty.

coefficients. More specifically, the negative error is further slightly reduced, while the positive error is increased. The origin of this increase can be traced back to the substantial dependence on μ_F of the resummed predictions with non-zero $C^{(1)}_{ij\to t\bar{t}H,I}$ coefficients, manifesting itself at larger scales. However, even after the redistribution of the error between the positive and negative parts, the overall size of the scale error, corresponding to the size of the error bar, is reduced after resummation by around 10 (13)% at 13 (14) TeV with respect to the NLO uncertainties. The scale error of the predictions is still larger than the pdf error of the NLO predictions which is not expected to be significantly influenced by the soft gluon corrections.

After our results for the absolute threshold resummation of the $pp \to t\bar{t}H$ become publicly available [16], related work appeared [25] that addresses the problem of soft gluon corrections to this process at NNLO accuracy. Both contributions target the same class of enhanced higher order correction. There are, however, several differences in the approximations applied, and in the treatment of theoretical uncertainty. First, we obtain the soft gluon corrections assuming the absolute threshold approximation and in Ref. [25] the soft gluon corrections are calculated for the differential invariant mass distribution of the $t\bar{t}H$ system, i.e. in the limit of the invariant mass approaching the partonic $\hat{s}$. Next, our paper relies on the classical Mellin resummation technique while the Soft-Collinear Effective Theory (SCET) is used in [25]. Although in the same threshold limit the two approaches formally resum the same set of logarithmic corrections, their actual treatments in the moment space and in the momentum space can lead to additional differences for hadronic cross sections, which, however, are not expected to be significant [26]. The two approaches differ also in the treatment of higher order corrections beyond the NNLO (in the fixed order perturbative QCD expansion): in [25] the NNLL resummed formula is expanded and truncated at the NNLO, whereas we perform the all-order resummation at NLL. In both the approaches the hard function containing information on off-shell dynamics is treated at the NLO accuracy (the case of a non-zero $C^{(1)}$ coefficient in our work) but the level of kinematical dependence of the corresponding expressions varies. In particular, in the absolute threshold resummation the value of the $C^{(1)}$ coefficient is calculated in the limit $\beta \to 0$. Finally, a significant difference in the treatment of the scale of the process and the related uncertainties takes place. We studied in detail the theoretical uncertainty of the improved cross-section due to independent variation of the renormalisation and the factorisation scale, leading to a conservative estimate of the theoretical uncertainty as compared to the equal scale case. Ref. [25] provides results for the renormalization scale equal to the factorization scale. To summarize, the two attempts to use the soft gluon resummation to improve $pp \to t\bar{t}H$ cross-section are complementary, indicating directions on how to further improve theoretical understanding and accuracy of both Mellin and SCET resummations.

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