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VOLUNTARY AND FORCED REDISTRIBUTION UNDER DEMOCRATIC RULE

There is a wide literature on the problem of division of goods from a perspective of both social choice theory and game theory. The former scrutinizes formal properties of different allocation rules (usually referred to as competing definitions of ‘distributive justice’)¹, while the latter usually concentrates on the interaction of strategies employed by actors striving to achieve the best preferred division of goods in question. It should be stressed that ‘best preferred’ is not necessarily tantamount to ‘self-interest maximizing’. Indeed, as the ample empirical evidence shows, the standard game-theoretical assumption of individual egoism does not hold true in some of laboratory games played by human subjects. In recent years we have been witnessing a rapid growth of experimental research conducted to test for alternative explanations of distributive behavior, such as *altruism or spitefulness* (Levine 1997), considerations for *fairness* or *reciprocity* (Fehr Schmidt 1999; Tyran Sausgruber 2002; Bolton Ockenfels, forthcoming), or *empathic responsiveness* (Fong 2003). On the other hand, self-interest was found to be of utmost significance in experiments where subjects’ payoffs were dependent on their own effort/productivity (Rutström Williams 2000, Gächter Riedl 2002), as well as in some games with random entitlements, e.g. hawk-dove game (Neugebauer Poulsen Schram 2002).

Games typically used to model an intentional division of goods are various types of dictator, ultimatum, and gangster games. They all refer to the problem of ‘splitting the cake’: dictator game assumes that one player, who is initially endowed with entire cake, is absolutely free to define the ultimate split between himself and the other; in ultimatum the receiver also has a say in that he can either accept or reject the proposal – in the

¹ For a brief overview of different distributive justice principles see Lissowski 2001, pp. 29–38.

latter case a proposer loses his entire initial endowment while a receiver gets nothing; and a gangster game is an explicit reversal of a dictator – it's the receiver who takes ultimate decision on how much to take from the initial owner of the cake. Dictator and gangster games have also been combined to form a democracy game, where a number of 'haves' and 'have-nots' vote over the final split of the pie. In this kind of game both forced and voluntary redistribution is brought about by a political decision procedure.

Redistribution game with charity transfers

The aim of the article is to propose a redistribution game, well-fit to distinguish between voluntary and politically-enforced tax transfers, as well as to model simple dynamics of such a twofold redistributive process.

The initial distribution of payoffs is exogenous to the game, though it may rightly be thought of as resulting from free-market labor contracts. Except for differences in amount of their initial earned incomes, agents are equal with regard to the type of decision each of them is to make. First, they take part in voting procedure on equal terms. Second, they may dispose their after-tax money in any manner suitable to them by keeping an arbitrary share of their income to themselves, and spreading the rest of it to the others (given they did not keep the entire sum to themselves).

Redistribution through tax system is forced in a sense that once a tax-rate is decided upon, agents are forced to pay a given percentage of their income, irrespectively of their own opinions on the right level of taxation. The presence of coercive element in tax collection renders it necessary to reserve some money for covering the cost of executing taxes from the reluctant. The cost of taxation (C) is defined as a given percentage of total tax revenues, and it may assume any value between 0% and 100% (or more conveniently between 0 and 1). $C = 0$ would imply absence of any executive cost, and $C = 1$ would amount to all-prodigal system in which the whole tax revenues are used exclusively to defray the costs of their collection.

Knowing an exact executive cost C , each player is called upon to cast a personal tax vote (t_i), which likewise may assume any value between 0 and 1. Then a linear tax-rate T is determined by a democratic rule as an average of all players' proposals:

$$T = \frac{1}{n} \sum_{i=1}^n t_i$$

The sole dedication of fiscal system in our model is diminishing the existing payoff inequalities. After collecting income taxes proportional to agents' initial earnings and pooling them into the budget, the executive cost C is subtracted, and the remaining sum is equally divided among the players. Two extremes would be $T = 0$, i.e. a *laissez-faire* system in which nobody pays any taxes whatsoever and each player is left with his initial payoff, and $T = 1$, i.e. a *strict egalitarian* system, where all incomes are taken away from the agents and subsequently divided equally between them. Obviously both *laissez-faire* and *strict egalitarian* systems can come about only as a result of all players voting 0 or 1, respectively.

If we denote player i 's initial payoff as p_i , then his after-tax payoff p'_i , allowing for actual difference between tax paid and subsidy received, can be computed as follows:

$$p'_i = (1 - T)p_i + T(1 - C)\bar{p}$$

where \bar{p} is a mean primary payoff over all players.

Holding tax-rate and cost constant, player i 's after-tax payoff p'_i depends partly on his own initial income p_i (the first summand) and partly on a mean primary payoff of a group \bar{p} (the second summand). After-tax incomes for some characteristic combinations of cost and tax level are juxtaposed in Table 1.

Table 1

Player i 's after-tax payoff

		Cost of execution		
		0.0	0.5	1.0
Tax-rate	0.0	p_i	p_i	p_i
	0.5	$\frac{1}{2}p_i + \frac{1}{2}\bar{p}$	$\frac{1}{2}p_i + \frac{1}{4}\bar{p}$	$\frac{1}{2}p_i$
	1.0	\bar{p}	$\frac{1}{2}\bar{p}$	0

As it was already mentioned, in *laissez-faire* system ($T = 0$) there is no redistribution and all agents retain their primary payoffs, while in *strict egalitarian* system ($T = 1$) all players receive the same amount, dependent on the average initial income and cost of tax execution.

Now if we take into account the difference between the initial and after-tax payoff, it can be noted that as a matter of fact the tax system is not linear. All transfers to and from budget included, real lump-sum of a tax paid by player i under tax-rate T is given by the formula:

$$\tau_i(T) = p_i - p'_i = T(p_i - (1 - C)\bar{p})$$

For any T , all agents whose relative initial payoff (p_i/\bar{p}) is higher than $1 - C$ pay positive taxes ($\tau_i > 0$), while those whose relative initial payoff is below that threshold receive net subsidy from the budget ($\tau_i < 0$). People initially earning exactly $(1 - C)\bar{p}$ can neither gain nor lose from the tax redistribution². Given this, it is trivial to determine how self-interested players should vote in order to establish linear tax-rate T that would maximize their after-tax payoffs. The whole society of players essentially splits into two groups, which for convenience reasons we shall call ‘rich’ ($\frac{p_i}{\bar{p}} > 1 - C$) and ‘poor’ ($\frac{p_i}{\bar{p}} < 1 - C$). It is in direct interest of the rich to vote for zero-percent tax, while in the direct interest of the poor it is to opt for 100% redistribution. The ‘intermediate class’ has no interest at all in any concrete tax-rate. To avoid random voting, we may assume that all agents having maximized their own after-tax income, in the second place vote to maximize an average after-tax income of a community as a whole. This would lead ‘intermediates’ to vote in line with a non-redistribution principle self-interested voting scheme, in which we shall make a point of reference for further analysis, is presented in Figure 1.

It is clear that as the cost increases, the threshold value for relative initial income p_i/\bar{p} drops from 1 (for $C = 0$) to 0 (for $C = 1$). Thus for any given initial distribution of income a number of players championing a complete redistribution is a non-decreasing function of C . As the cost is approaching its absolute maximum at 1, the tax-rate T established by a popular vote in a society of self-interested players will be closer to 0. In an extreme case where $C = 1$, virtually no agent can gain from tax redistribution, and a laissez-faire system must prevail. Generally, we will refer to the tax-rate T established in a society of egoistic players as to the *Polarized Voting Tax (PVT)*³.

As opposed to taxes, redistribution through individual charitable transfers to other players involves no costs as there is no need to compel people to do what they are willing to do of their own initiative⁴. Thus if we

² This may be seen as an exemplification of positive/negative income tax, advocated by Milton and Rose Friedman (1996, pp. 114–119).

³ Similar redistribution mechanism to the one described above, though not allowing for voluntary charity transfers, was incorporated by Elizabeth Jean Wood (1999) into her model of rapid social change. Prof. Wood defined C to be an increasing function of T , and analyzed the model from a point of view of the decisive voter.

⁴ To be sure, it is simplifying assumption as private charity also incurs cost of collecting and distributing donations. However, to justify this feature of our model it is enough to notice that, as empirical evidence shows, the cost of private charity is substantially lower than in state-administered system (West, Ferris, 1999).

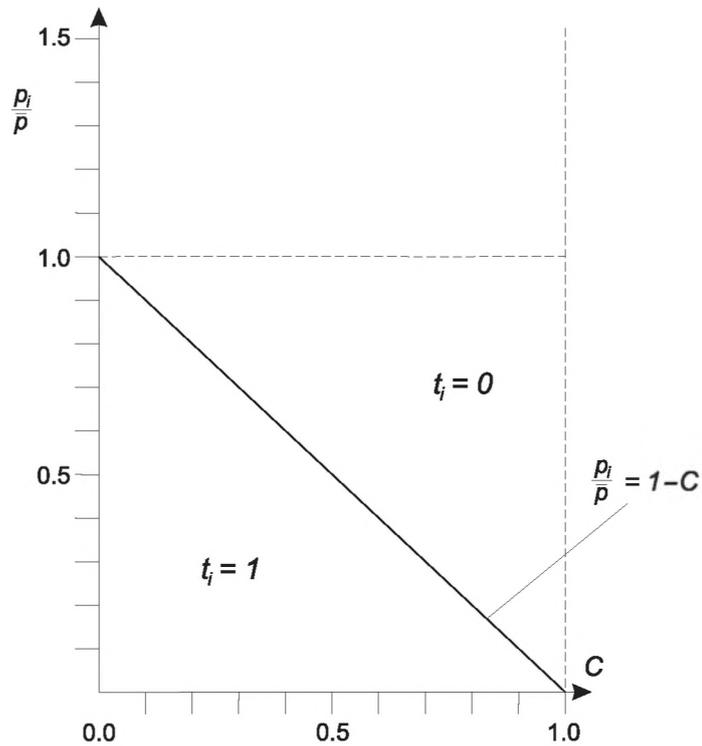


Figure 1. Polarized voting

denote charity transfer player i gives away as \tilde{c}_i , and charity transfer player j receives as \tilde{c}_j , we can state that:

$$\sum_{i=1}^n \tilde{c}_i = \sum_{j=1}^n \tilde{c}_j$$

In the following for simplicity reasons we will focus our attention on the redistribution process in a dyadic society with one rich and one poor player. However, it should be noted that this restriction obviously stops us from investigating some distinct new qualities of the game that emerge as number of agents exceeds 2.

Game with 2 players

Let us start with presenting a one-shot normal form redistribution game for two players. Suppose that players have different initial payoffs, and cost of execution satisfies following condition:

$$C < \frac{p_R - p_P}{p_R + p_P}$$

where p_P and p_R stand for poor and rich player's payoffs respectively.

The condition warrants that it is in the self-interest of the poor player to vote for 100% tax-rate which will result in establishing *Polarized Voting Tax (PVT)* equal to 50%. If cost of the execution exceeded the threshold value, even a poor player would incur losses at any positive tax level and therefore would be inclined to vote 0%.

It is obvious that at non-zero cost of the tax execution rich agent cannot opt for anything but 0% tax. Even if he is an altruist willing to support a poor player, it is better for him to do it by direct charitable payment which does not involve any cost. It is also plausible to assume that no poor agent will be interested in passing part of his initial payoff to the rich as this would further augment the original gap between himself and the latter. Therefore, charity transfer poor player gives away to the other in no case exceeds zero. Under these assumptions, rich votes 0% and chooses between alternative amounts of charitable transfer, while the poor one gives no donation and chooses how to vote between alternative tax-rates. Resulting from global tax-rate, as it is half the tax-rate proposed by the poor, varies between 0 and 50% (*PVT*).

With the help of the example suppose that initially a rich player earns \$25 and poor \$15, while the cost of execution is 10%. Suppose further that the rich considers giving \$2 to support the worse-off player, and the latter takes into account voting either 0 or 100%. Such a game can be presented in the following table:

Table 2

Redistribution game in normal form

		Poor	
		$t_P = 0 \rightarrow T = 0.0$	$t_P = 1 \rightarrow T = 0.5$
Rich	$\hat{c}_R = 2$	23.00	19.50
	$\hat{c}_R = 0$	25.00	21.50

Both people have dominant strategies: regardless of poor player's choice it is better for the rich player not to make any donation, and regardless of rich player's behavior it is more profitable for the poor to establish maximum 50% tax-rate by voting 100%. Thus, the equilibrium solution is for the rich

player not to contribute anything, and for the poor to vote for maximum redistribution possible (in which case the rich person's final payoff is \$21.50, whereas for the poor it is \$16.50). This, however, is not a Pareto-optimal result as for both players, it would be more profitable, had the rich chosen to donate \$2, and the poor decided to opt for a laissez-faire system (then the rich would have \$23 and the poor \$17). Thus, a redistribution game turns out to be an asymmetric variant of prisoner's dilemma.

The surplus players can divide between themselves the results from reduction of execution cost due to the virtual elimination of tax system. Holding players' initial payoffs constant, the lump-sum of the surplus depends on the actual tax-rate and the cost of execution.

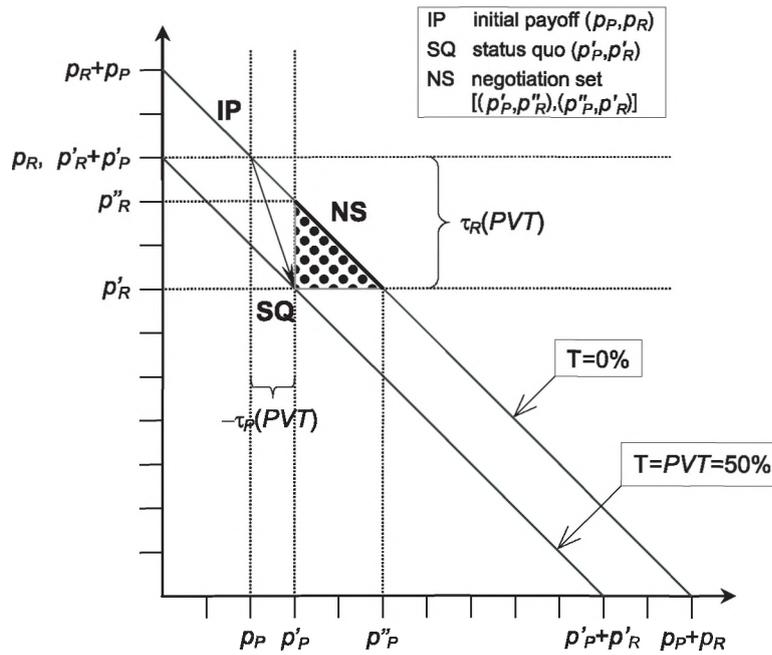


Figure 2. Negotiating surplus

In Figure 2 we see feasible outcomes of the game if players are in position to conclude a binding contract (poor and rich agent's payoffs at horizontal and vertical axes, respectively). They start the game with their initial payoffs (IP) when the rich earns p_R and the poor p_P . Under a democracy rule, a poor player can redistribute some wealth from the rich to himself by raising tax-rate up to 50% (PVT). By-product of tax redistribution is shifting of the budget constraint down and to the left. The shift occurs because at $T = 0.5$ some part of players' total payoff is consumed by the executive cost. Now

we may consider a point (p'_P, p'_R) to be status quo (SQ) since it is a pair of incomes that each agent is able to assure himself of his own, regardless of the other's behavior. At SQ the rich pays lump-sum tax equal to $\tau_R(PVT)$, and the poor receives a net subsidy equal to $-\tau_P(PVT)$. The difference between the two is the amount that could be re-gained by establishing laissez-faire system. However, the poor has no direct interest in lowering tax-rate for it would shift the outcome back in direction of IP , thus reducing his final payoff. On the other hand, 50% tax-rate brings harm to the rich player who not only covers the subsidy to the poor but also defrays the entire executive cost. Thus, it would be much to his interest to replace a politically-forced costly redistribution with a voluntary cost-free charity transfer to the other. To encourage the poor to vote for 0% tax-rate, though, the rich player needs to offer him a lump-sum at least equal to the loss incurred by the poor from eliminating tax redistribution, i.e. $-\tau_P(PVT)$. The thick black line in Figure 2 indicates the negotiation set, i.e. a number of solutions to the problem of how the surplus gained from abolition of tax system should be divided between the players. If entire surplus goes to the rich, the ultimate outcome will be (p'_P, p''_R) , if it falls solely to the poor, the outcome will be (p''_P, p'_R) . All combinations of payoffs between those two points are feasible as well.

Dynamical substitution between voluntary and forced redistribution

At this point we shall introduce dynamics into the system. Suppose the game is infinitely iterated, with initial payoffs and the cost of tax execution held constant and known to the players who have no possibility of direct communication. Each round will consist of the following sequence of moves: first players vote on redistribution, then tax transfers to and from the budget take place according to the current tax-rate, and finally it is up to players to give away some part of their income to the other.

At the outset poor player, willing to secure to himself the status quo outcome, votes for maximum redistribution and thus PVT at 50% is established. Now the rich player has an occasion to signal his willingness to replace tax redistribution with voluntary transfer by offering a donation to the poor. The lump-sum of this first donation depends on rich agent's *charitable initiative*. In the second round the poor player may react to the donation by reducing his demand for tax redistribution according to his personal *demanding attitude*. In turn the rich player reacts to

tax-rate decrease according to his *generosity*⁵, and so forth the process continues ad infinitum. Let us now formally define the individual features of players:

- ε – *charitable initiative*, is an amount of income, expressed as a share of $\tau_R(PVT)$, that a rich player is willing to give away directly to the other in the first round of the game; ε is effective as long as it satisfies the condition $\varepsilon \leq \frac{p_R - \tau_R(PVT)}{\tau_R(PVT)}$; greater values of ε are cut down to that threshold level, for donation cannot exceed rich agent's entire income.
- γ – *generosity*, is a share of $\tau_R(PVT)$ that a rich player is willing to donate under laissez-faire system; effective γ 's obey $\gamma \leq \frac{p_R}{\tau_R(PVT)}$.
- δ – *demanding attitude*, is a share of $-\tau_P(PVT)$ that a poor player will demand as a compensation for eliminating tax redistribution completely⁶.

Algorithm of decision making in each round of the game may be summarized in the table 3.

As we see in each round the poor player gives no donation ($\hat{c}_P = 0$), and the rich player votes for 0% tax-rate ($t_R = 0$). From the second round on tax-vote by the poor agent depends on his personal demanding attitude γ and the donation received in the preceding round. If at time $i - 1$ he received no charity transfer, at time i he will vote 100% (except for $\delta = 0$, in which case he votes 0% regardless of donation received); if transfer equaled $-\delta\tau_P(PVT)$ or more, he will be inclined to vote 0% (t_P 's lower limit is obviously 0, so lower values are automatically increased to this level); if donation was somewhere between 0 and $-\delta\tau_P(PVT)$, he will depart from 100% vote proportionately. It is safety strategy for the poor to have $\delta \geq 1$. In case $\delta \geq 1$, he is ready to vote 0% only if rich agent fully covers the loss incurred by the poor from abandoning the status quo.

On the other hand, rich agent's donation depends on his individual generosity γ and tax-rate T in the current round. If tax-rate is PVT , he

⁵ These three features (parameters) define a type of agent, though they are activated contextually, i.e. for a player in poor position only demanding attitude is relevant, whereas for a player in rich position it is charitable initiative and generosity. It should also be stressed that we use the terms in neutral sense and attach no moral value, neither positive nor negative, to any of the features. For instance, 'generous' actions may as well be motivated by strict self-interest.

⁶ Negative sign before $\tau_P(PVT)$ is due to the fact that at PVT poor player by definition 'pays' negative tax (which means that actually he receives net subsidy from the budget).

Table 3

Dynamics of redistribution game⁷

		Poor player	Rich player
Round no. 1	Voting	$t_P = 1$	$t_R = 0$
	Donation	$\hat{c}_P = 0$	$\hat{c}_R = \varepsilon \tau_R(PVT)$
Round no. 2 and next	Voting	$t_P = \begin{cases} 0 & \delta=0 \\ 1 + \frac{\hat{c}_P}{\delta \tau_P(PVT)} & \delta>0 \end{cases}$ * $\hat{c}_P = \hat{c}_R$ of the preceding round	$t_R = 0$
	Donation	$\hat{c}_P = 0$	$\hat{c}_R = \begin{cases} 0 & PVT=0 \\ \frac{PVT-T}{PVT} \cdot \gamma \tau_R(PVT) & PVT>0 \end{cases}$ * T of the current round

gives no donation, regardless of his generosity; if T is 0%, his donation equals $\gamma \tau_R(PVT)$; if T is somewhere between 0 and PVT , he offers a proportionate charity transfer. His safety strategy is to have $\gamma \leq 1$. In case $\gamma = 1$, he is ready to donate to the poor entire surplus resulting from abolishing tax redistribution. Was his generosity greater than that, he would expose himself to the risk of earning less than status quo.

Under what conditions can tax redistribution be completely replaced with voluntary transfers on part of the rich? To answer this, let us find levels of ε , γ , and δ that make it possible to establish laissez-faire system. Roughly speaking, to make a tax-rate go down to zero, it is necessary that the rich was generous *enough*, whereas the poor was not demanding *too much*. A threshold value of poor's demanding attitude (δ) as a function of rich's generosity (γ) is given by the *correspondence formula*⁸ (see Appendix for details):

$$\delta = -\frac{\tau_R}{\tau_P} \cdot \gamma$$

⁷ The same algorithm may be applied to multiplayer game. Tax-vote by poor player depends on the donation he received in the previous round, though when number of players exceeds two, the donation need not be equal to the transfer made by any particular rich agent (one possible way to distribute charity transfers among the poor is to employ leximin principle). The 'intermediate class' could be defined to have $t_I = 0$ and $\hat{c}_I = 0$ for each round.

⁸ The visual presentation of the equation will be referred to as *correspondence line*. To avoid excessive notation, from now on τ_R and τ_P will denote lump-sum tax paid at PVT rate.

If δ satisfies the equation than substitution rate between taxes and free donations is exactly the same for both agents. It means that in order to completely eliminate taxes the rich player is ready to give away the amount that is precisely as much as poor player demands for reducing his tax-vote to zero. If δ exceeds the threshold value, tax-rate T will be equal to PVT from the very beginning, or will be approaching limit at PVT with speed negatively correlated to the charitable initiative of the rich (ε). If δ is less than that, at some point of the game laissez-faire system will be established (the smaller δ and the larger ε , the sooner tax-rate will fall to zero). If δ is exactly equal to the threshold value, than the tax-rate established in the second round will hold for the rest of the game. Thus ultimate tax-rate in this case is determined by ε and may assume any value between 0 and 1. Its exact value is given by (see Appendix):

$$T = \frac{1}{2} \left(1 - \frac{\varepsilon}{\gamma} \right), \quad \gamma > 0, \varepsilon \leq \gamma$$

Payoff structure at time approaching infinity

To conclude the analysis we will examine limit payoffs, as time approaches infinity, for different combinations of generosity γ and demanding attitude δ , while holding charitable initiative ε equal to γ ⁹. Next we will point pairs of γ and δ that form Nash equilibria. To make the analysis easier to follow we will plot the results on the same diagram as seen in Figure 3.

Let us denote the rich agent's status quo payoff (PVT , no donation) as $R0$, payoff better than that as $R+$, and worse one as $R-$; and respectively for poor agent: $P0$, $P+$, $P-$. In Figure 4 we show how given combinations of γ and δ prove better/worse than status quo for either player.

If generosity γ of the rich agent is too small in comparison to demanding attitude δ of the poor, the tax-rate will approach a limit at PVT , and thus both players will receive their status quo payoffs $R0$, $P0$ (entire area above correspondence line). However, if γ and δ are kept in 'reasonable' proportion to each other, a lower tax-rate is established, and there is a surplus resulting

⁹ For a rich agent playing his safety strategy ($\gamma \leq 1$), it is always profitable to have laissez-faire system established. As we focus on the limit payoffs at infinity, single first round charity expense determined by ε does not affect rich agent's payoff, so he does not incur any more risk by giving ε value equal to (or even higher than) δ . And by doing so he is able to reduce tax-rate to zero in case γ and δ happen to lie on the correspondence line.

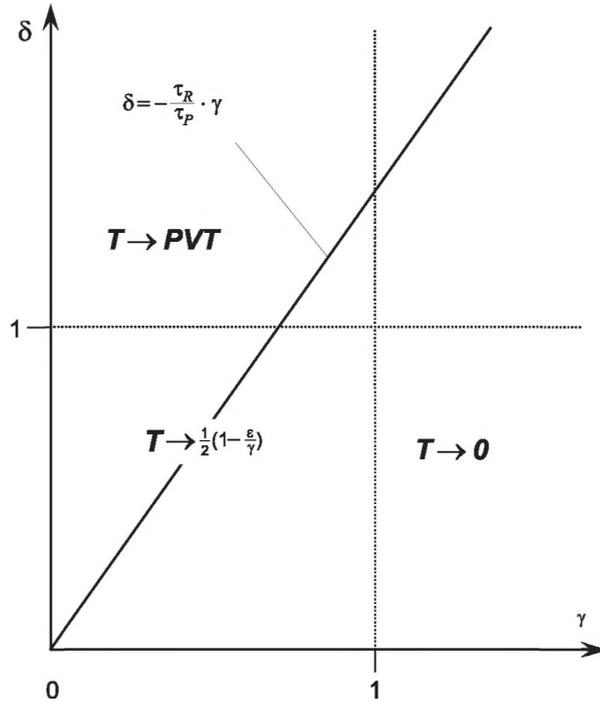


Figure 3. Generosity, demanding attitude and tax-rate

from cutting down on execution costs. Thus on and below the correspondence line always at least one of the agents is better-off than in status quo. It's worth noticing that for any given value of γ both agents' limit payoffs are independent of δ , as far as δ does not exceed the threshold value¹⁰. Particularly, for $\gamma = -\tau_P/\tau_R$ entire surplus is taken by the rich, while the poor is left with his status quo earning, and for $\gamma = 1$ entire surplus goes to the poor, while the rich keeps his status quo income. Generally, for all points on and below the correspondence line, as γ coordinate is reduced, the rich agent's payoff increases 'at the expense' of the poor.

We may now see that when considering limit payoffs redistribution game is basically a variant of ultimatum game. The rich agent proposes a donation depending on his generosity γ , and the poor agent either accepts the offer or rejects it depending on his demanding attitude δ . In case δ lies above a correspondence line and the proposition is rejected, both players are left with their status quo payoffs. In contrast to the original ultimatum game

¹⁰ The reason for it is that parameter γ by its very definition determines the lump-sum that goes to the poor by means of voluntary transfer under laissez-faire system, and as we know from Figure 3 the limit tax-rate under correspondence line is zero.

Voluntary and Forced Redistribution under Democratic Rule

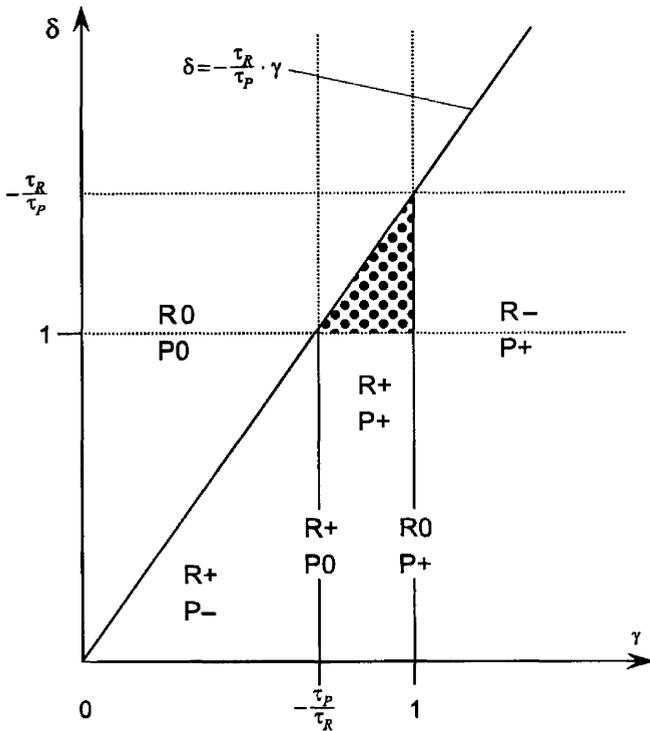


Figure 4. Limit payoffs in comparison to status quo

where players' payoffs could not fall below zero, reaching an 'agreement' in the redistribution game may lead to one of the players towards being materially worse-off than in status quo. The latter situation is possible when either player does not play his safety strategy.

As we may read from the diagram, safety strategy for the poor player is to have $\delta \geq 1$. Was δ below 1, the poor could suffer loss in comparison to status quo, if the rich had his γ below $-\tau_P/\tau_R$. Similarly it is safe for the rich to have $\gamma \leq 1$. Was his generosity greater than that, his limit income could fall short of status quo, in case the poor was not too demanding ($\delta \leq -\tau_R/\tau_P$). The dotted triangle indicates pairs of safe γ and δ that bring profit to both sides (except for point $(-\tau_P/\tau_R, 1)$ and the right side of the triangle, where only one player gains, while the other stays with status quo payoff). All γ - δ pairs in the triangle (and rectangle below it as well) lead to payoffs that belong to the negotiation set presented in Figure 2.

Further scrutiny leads us to the conclusion that for a self-interest maximizing poor player $\delta = 1$ is a dominant strategy. Such a choice is the analogue of the receiver accepting zero-share in ultimatum game with continuous payoffs.

Nash equilibria of the redistribution game

Finally, let us point out combinations of γ and δ that form Nash equilibria of the redistribution game. First of all, we may rule out all points that violate either agent's safety level, i.e. $\gamma > 1$ or $\delta < 1$. Further let us consider γ belonging to $(-\tau_P/\tau_R, 1]$. The poor player's best response to any given γ in the range is **any** δ less or equal to threshold value. However, rich agent's best response for any δ less or equal to $-\tau_R/\tau_P$ is choosing γ in such a way as to locate the point (γ, δ) on the correspondence line. Thus all points on a hypotenuse of a dotted triangle in Figure 4 are Nash equilibria (thick line in Figure 5), while the others are not¹¹.

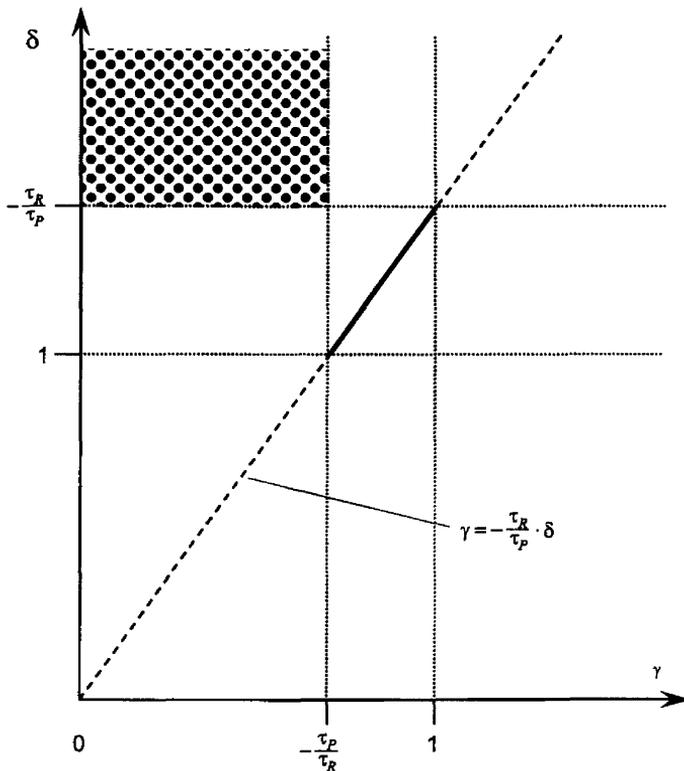


Figure 5. Nash equilibria (for $\varepsilon = \delta$)

¹¹ As $\delta = 1$ is a poor player's dominant strategy, combination of $\delta = 1$ and $\gamma = \varepsilon = -\tau_P/\tau_R$ is a unique solution of the redistribution game for strictly self-interested players. In this case poor player gets exactly his status quo income while the rich increases his status quo payoff by re-gaining the entire surplus resulting from abolition of executive costs of tax system.

For γ belonging to $[0, -\tau_P/\tau_R]$ any level of δ over the correspondence line may be considered the best response, as well as for δ equal to or greater than $-\tau_R/\tau_P$ the best response could be any γ to the left of the correspondence line. Thus Nash equilibria are also all points of the dotted rectangle, although they obviously are not Pareto-optimal. As empirical evidence from dictator and gangster games shows, it may well be the case that subjects actions lead to suboptimal outcomes. "The generous nature of individuals found in fairness games does not overcome the distribution struggle. While dictators are prepared to give up a part of their endowment, gangsters demand a much bigger share of the cake for themselves" (Eichenberger Oberholzer-Gee 1998, p. 196).

Graphical illustration of system dynamics

To give an example of redistributive dynamics we will conclude the article with a few characteristic cases of system evolution. In each case agents' initial payoffs are \$90 (rich) and \$10 (poor), whereas the cost of tax execution is 20%. At *PVT* (50%) the rich agent's payoff is equal to \$65 and the poor player earns \$25. On each graph a line - - - refers to the rich agent's final income (all taxes and voluntary transfers included), a line _ _ _ _ to the poor agent's income, and a line _ _ _ _ reports the history of the tax-rate.

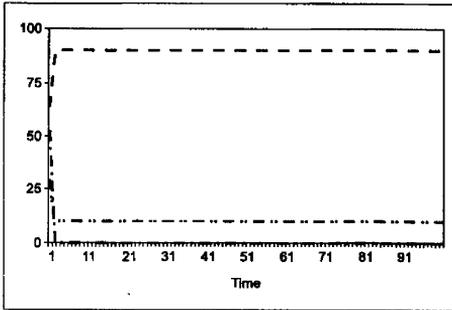
Illustration no. 1 shows how a stable state is immediately reached if the poor agent exhibits no demand for redistribution ($\delta = 0$). If at the same time the rich agent exhibits no generosity ($\gamma = 0$), he takes entire surplus resulting from eliminating executive costs, and both players earn their initial payoffs. It may be seen as an instance of the poor player's high moral standards that hold him back from exploiting the democratic procedure to enforce more profitable income distribution. To be sure, that sort of consideration would not even pass through the mind of *homo oeconomicus*.

Illustration no. 2 shows that if the poor agent's demanding attitude is anything apart from zero, it is impossible to establish laissez-faire system without charitable initiative of the rich ($\varepsilon = 0$) - no matter how great his generosity could be ($\gamma \geq 0$). The tax-rate does not deviate from *PVT* even by the smallest margin and in each round the agents earn their status quo payoffs (\$65 and \$25).

Graphs nos. 3 and 4 show the history of reaching Pareto-optimal distribution (with both agents' payoffs higher than status quo). In no. 3 the rich player does not play his best response. By reducing his generosity and

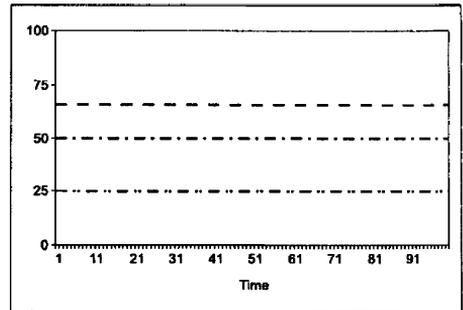
1. No demand for redistribution

$\gamma=0.00 \quad \varepsilon=0.00 \quad \delta=0.00$



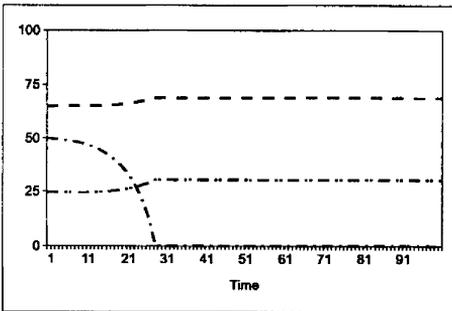
2. No charitable initiative (suboptimal NE)

$\gamma \geq 0.00 \quad \varepsilon=0.00 \quad \delta > 0.00$



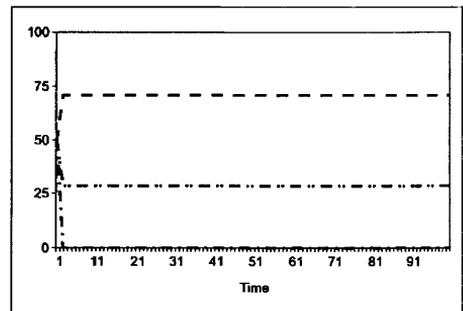
3. Reaching Pareto-optimal outcome (not NE)

$\gamma=0.85 \quad \varepsilon=0.01 \quad \delta=1.25$



4. Pareto-optimal Nash Equilibrium

$\gamma=0.75 \quad \varepsilon=0.75 \quad \delta=1.25$

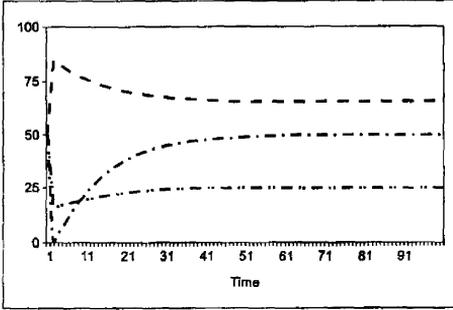


raising his charitable initiative appropriately, he could assure himself a higher payoff, taking benefit of the poor agent's moderate demand for redistribution. If he did so, Nash equilibrium presented in graph no. 4 would be established: $(\gamma = 0.75, \delta = 1.25)$ is a point lying exactly on the correspondence line.

Graphs nos. 5 and 6 show how system can recede to *PVT* after initial reduction of tax-rate to nigh-zero level. No. 5 illustrates that it is impossible to dupe the poor player into laissez-faire system by substantial charitable initiative combined with low generosity. Even though the poor agent plays δ far below his safety strategy, the tax-rate after the initial reduction gradually recedes to *PVT*. On the other hand, in no. 6 poor player's demand for redistribution is too much even for a super-generous rich agent. In spite of large initial charity transfers (that even made the recipient wealthier

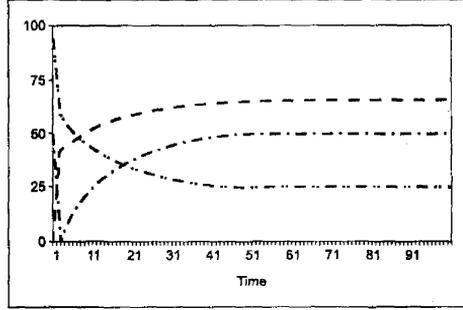
5. Recession to PVT due to insufficient generosity

$\gamma=0.20 \quad \varepsilon=0.40 \quad \delta=0.36$



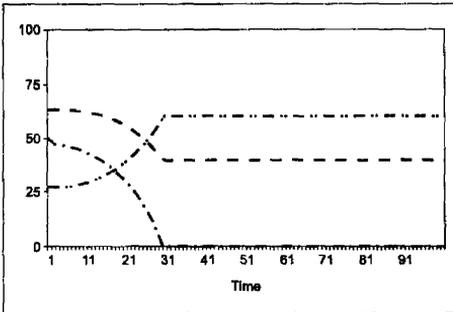
6. Recession to PVT due to excessive demand

$\gamma=2.00 \quad \varepsilon=2.50 \quad \delta=3.60$



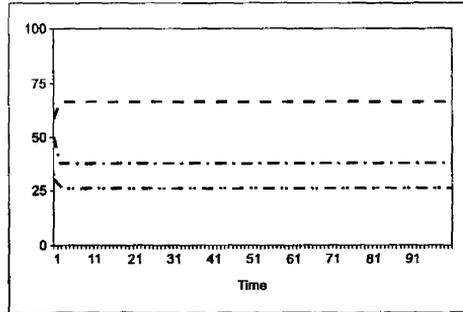
7. Super-generosity

$\gamma=2.00 \quad \varepsilon=0.10 \quad \delta=3.00$



8. Insufficient charitable initiative

$\gamma=0.80 \quad \varepsilon=0.20 \quad \delta=1.33$



than his benefactor), the poor player departs from voting 0% and *PVT* is gradually brought back.

Illustration no. 7 depicts a rich man who is willing to give away most of his income, starting with a small initial contribution. The poor agent's demanding attitude is below the threshold value, so the tax-rate is being gradually reduced to zero. The poor agent takes benefit of the rich agent's generosity, receiving a substantial part of his wealth, and ultimately becoming richer than the donor.

Finally, at no. 8 we see a game between two players whose generosity and demanding attitude are exactly corresponding to each other. However, due to insufficient charitable initiative on the part of the rich, the tax-rate is fixed at 37.5% and benefits from complete eliminating executive costs are lost.

Concluding remarks

The purpose of the article was to model a redistribution process, allowing for the interplay between transfers forced by means of the tax system and voluntary donations to the worse-off. The dynamics of the system were guided by agents' personal features, namely charitable initiative and generosity on part of the rich, and demanding attitude on part of the poor. We have shown that even under the assumption of exclusive self-interest seeking, there are Pareto-optimal Nash equilibria that result in a complete substitution of free charity for tax redistribution, as well as suboptimal equilibria that keep the volume of tax redistribution intact. It is a question of empirical research whether real-life subjects are able to find their way to elimination of excessive cost of politically forced transfers. It is also a matter for further discussion how dynamics of the game are affected by introducing greater number of players and focusing attention on discounted payoffs rather than looking at the limit distribution of income.

Appendix

To simplify notation let the variables over time be denoted as:

- c_i – voluntary transfer from rich to poor player in round i ,
- t_i – poor agent's tax-vote in round i ,
- T_i – tax-rate in round i ,

Constants (as described in the text):

$$p_R, p_P, \bar{p}, \tau_R, \tau_P, C, \varepsilon, \gamma, \delta$$

At time i player k 's payoff is given by formula:

$$(1) \quad p_k(T_i) = (1 - T_i)p_k + T_i(1 - C)\bar{p}$$

According to decision algorithm (see Table 3), rich agent's donation in first round is given by:

$$(2) \quad c_1 = \varepsilon\tau_R$$

His donations in round $i \geq 2$ are:

$$c_i = \frac{PVT - T_i}{PVT} \gamma\tau_R.$$

Since $PVT = \frac{1}{2}$, we get

$$(3) \quad c_i = (1 - 2T_i)\gamma\tau_R.$$

Tax-rate at time i is always half the tax-rate proposed by the poor (again see Table 3):

$$(4) \quad T_i = \frac{1}{2}t_t = \frac{1}{2} \left(1 + \frac{C_{i-1}}{\delta\tau_P} \right)$$

Substituting (2) into (4), we obtain tax-rate in second round:

$$T_2 = \frac{1}{2} \left(1 + \frac{\varepsilon\tau_R}{\delta\tau_P} \right).$$

Substituting (3) into (4), we obtain difference equation for $i \geq 3$:

$$T_i = -\frac{\gamma\tau_R}{\delta\tau_P}T_{i-1} + \frac{1}{2} \left(1 + \frac{\gamma\tau_R}{\delta\tau_P} \right).$$

To make it easier to handle let us rewrite it as:

$$(6) \quad T_i = aT_{i-1} + b,$$

where $a = -(\gamma\tau_R/\delta\tau_P)$, $b = \frac{1}{2}(1 - a)$.

Or, alternatively:

$$(7) \quad T_i = a(T_{i-1} - \frac{1}{2}) + \frac{1}{2}.$$

Now the sequence of T_i 's for $i \geq 3$ is:

$$T_3 = aT_2 + b$$

$$T_4 = aT_3 + b = a^2T_2 + ab + b$$

\vdots

$$T_i = a^{i-2}T_2 + a^{i-3}b + a^{i-4}b + \dots + ab + b = a^{i-2}T_2 + \frac{1-a^{i-2}}{1-a}b = a^{i-2}T_2 + \frac{1}{2}(1 - a^{i-2}) = a^{i-2} \left(T_2 - \frac{1}{2} \right) + \frac{1}{2}$$

Thus for $i \geq 3$:

$$(8) \quad T_i = a^{i-2} \left(T_2 - \frac{1}{2} \right) + \frac{1}{2}.$$

Solving (8) with $i \rightarrow \infty$ we get:

1. $0 \leq a < 1$:

$$a^{i-2} \rightarrow 0 \text{ and } \lim_{i \rightarrow \infty} T_i = \frac{1}{2}$$

2. $a = 1$:

$$T_i = T_2 \quad \forall i$$

3. $a > 1$:

if $T_2 = \frac{1}{2}$ (which implies $\varepsilon = 0$ and $\delta > 0$): $T_i = \frac{1}{2} \quad \forall i$,

if $0 \leq T_2 < \frac{1}{2}$: $\exists m \geq 2$ for which $T_m \leq 0$ (since in (8) the expression in parentheses is negative, and a^{m-2} increases in m). Since any T lower than zero is automatically increased to zero, we have $T_m = 0$.

Now let m be the smallest possible number of a round. In accordance with (7), if $T_m = 0$, then $T_{m+1} = (1 - a)/2 < 0$. Therefore $T_{m+1} = 0$, and since the same holds true for all subsequent rounds, $\lim_{i \rightarrow \infty} T_i = 0$.

Deciphering a , we get:

1. For $\delta > -\frac{\tau_R}{\tau_P} \cdot \gamma$, $\lim_{i \rightarrow \infty} T_i = \frac{1}{2}$, $\gamma > 0$. If $\gamma = 0$, $T_i = 0 \forall i \geq 2$ (see Table 3).
2. For $\delta = -\frac{\tau_R}{\tau_P} \cdot \gamma$, $\lim_{i \rightarrow \infty} T_i = T_2 = \frac{1}{2} \left(1 + \frac{\varepsilon \tau_R}{\delta \tau_P} \right)$, $\delta > 0$. Since $\delta = -\frac{\tau_R}{\tau_P} \cdot \gamma$, after transformation we receive: $\lim_{i \rightarrow \infty} T_i = \frac{1}{2} \left(1 - \frac{\varepsilon}{\gamma} \right)$, $\gamma > 0$. If $\delta = \gamma = 0$, $T_i = 0 \forall i \geq 2$ (see Table 3).
3. For $\delta < -\frac{\tau_R}{\tau_P} \cdot \gamma$,
 - a) if $\varepsilon = 0$ and $\delta > 0$, $\lim_{i \rightarrow \infty} T_i = \frac{1}{2}$;
 - b) if $\varepsilon > 0$, $\lim_{i \rightarrow \infty} T_i = 0$.

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VOLUNTARY AND FORCED REDISTRIBUTION UNDER DEMOCRATIC RULE

There is a wide literature on the problem of division of goods from a perspective of both social choice theory and game theory. The former scrutinizes formal properties of different allocation rules (usually referred to as competing definitions of ‘distributive justice’)¹, while the latter usually concentrates on the interaction of strategies employed by actors striving to achieve the best preferred division of goods in question. It should be stressed that ‘best preferred’ is not necessarily tantamount to ‘self-interest maximizing’. Indeed, as the ample empirical evidence shows, the standard game-theoretical assumption of individual egoism does not hold true in some of laboratory games played by human subjects. In recent years we have been witnessing a rapid growth of experimental research conducted to test for alternative explanations of distributive behavior, such as *altruism or spitefulness* (Levine 1997), considerations for *fairness or reciprocity* (Fehr Schmidt 1999; Tyran Sausgruber 2002; Bolton Ockenfels, forthcoming), or *empathic responsiveness* (Fong 2003). On the other hand, self-interest was found to be of utmost significance in experiments where subjects’ payoffs were dependent on their own effort/productivity (Rutström Williams 2000, Gächter Riedl 2002), as well as in some games with random entitlements, e.g. hawk-dove game (Neugebauer Poulsen Schram 2002).

Games typically used to model an intentional division of goods are various types of dictator, ultimatum, and gangster games. They all refer to the problem of ‘splitting the cake’: dictator game assumes that one player, who is initially endowed with entire cake, is absolutely free to define the ultimate split between himself and the other; in ultimatum the receiver also has a say in that he can either accept or reject the proposal – in the

¹ For a brief overview of different distributive justice principles see Lissowski 2001, pp. 29–38.

latter case a proposer loses his entire initial endowment while a receiver gets nothing; and a gangster game is an explicit reversal of a dictator – it's the receiver who takes ultimate decision on how much to take from the initial owner of the cake. Dictator and gangster games have also been combined to form a democracy game, where a number of 'haves' and 'have-nots' vote over the final split of the pie. In this kind of game both forced and voluntary redistribution is brought about by a political decision procedure.

Redistribution game with charity transfers

The aim of the article is to propose a redistribution game, well-fit to distinguish between voluntary and politically-enforced tax transfers, as well as to model simple dynamics of such a twofold redistributive process.

The initial distribution of payoffs is exogenous to the game, though it may rightly be thought of as resulting from free-market labor contracts. Except for differences in amount of their initial earned incomes, agents are equal with regard to the type of decision each of them is to make. First, they take part in voting procedure on equal terms. Second, they may dispose their after-tax money in any manner suitable to them by keeping an arbitrary share of their income to themselves, and spreading the rest of it to the others (given they did not keep the entire sum to themselves).

Redistribution through tax system is forced in a sense that once a tax-rate is decided upon, agents are forced to pay a given percentage of their income, irrespectively of their own opinions on the right level of taxation. The presence of coercive element in tax collection renders it necessary to reserve some money for covering the cost of executing taxes from the reluctant. The cost of taxation (C) is defined as a given percentage of total tax revenues, and it may assume any value between 0% and 100% (or more conveniently between 0 and 1). $C = 0$ would imply absence of any executive cost, and $C = 1$ would amount to all-prodigal system in which the whole tax revenues are used exclusively to defray the costs of their collection.

Knowing an exact executive cost C , each player is called upon to cast a personal tax vote (t_i), which likewise may assume any value between 0 and 1. Then a linear tax-rate T is determined by a democratic rule as an average of all players' proposals:

$$T = \frac{1}{n} \sum_{i=1}^n t_i$$

The sole dedication of fiscal system in our model is diminishing the existing payoff inequalities. After collecting income taxes proportional to agents' initial earnings and pooling them into the budget, the executive cost C is subtracted, and the remaining sum is equally divided among the players. Two extremes would be $T = 0$, i.e. a *laissez-faire* system in which nobody pays any taxes whatsoever and each player is left with his initial payoff, and $T = 1$, i.e. a *strict egalitarian* system, where all incomes are taken away from the agents and subsequently divided equally between them. Obviously both *laissez-faire* and *strict egalitarian* systems can come about only as a result of all players voting 0 or 1, respectively.

If we denote player i 's initial payoff as p_i , then his after-tax payoff p'_i , allowing for actual difference between tax paid and subsidy received, can be computed as follows:

$$p'_i = (1 - T)p_i + T(1 - C)\bar{p}$$

where \bar{p} is a mean primary payoff over all players.

Holding tax-rate and cost constant, player i 's after-tax payoff p'_i depends partly on his own initial income p_i (the first summand) and partly on a mean primary payoff of a group \bar{p} (the second summand). After-tax incomes for some characteristic combinations of cost and tax level are juxtaposed in Table 1.

Table 1

Player i 's after-tax payoff

		Cost of execution		
		0.0	0.5	1.0
Tax-rate	0.0	p_i	p_i	p_i
	0.5	$\frac{1}{2}p_i + \frac{1}{2}\bar{p}$	$\frac{1}{2}p_i + \frac{1}{4}\bar{p}$	$\frac{1}{2}p_i$
	1.0	\bar{p}	$\frac{1}{2}\bar{p}$	0

As it was already mentioned, in *laissez-faire* system ($T = 0$) there is no redistribution and all agents retain their primary payoffs, while in *strict egalitarian* system ($T = 1$) all players receive the same amount, dependent on the average initial income and cost of tax execution.

Now if we take into account the difference between the initial and after-tax payoff, it can be noted that as a matter of fact the tax system is not linear. All transfers to and from budget included, real lump-sum of a tax paid by player i under tax-rate T is given by the formula:

$$\tau_i(T) = p_i - p'_i = T(p_i - (1 - C)\bar{p})$$

For any T , all agents whose relative initial payoff (p_i/\bar{p}) is higher than $1 - C$ pay positive taxes ($\tau_i > 0$), while those whose relative initial payoff is below that threshold receive net subsidy from the budget ($\tau_i < 0$). People initially earning exactly $(1 - C)\bar{p}$ can neither gain nor lose from the tax redistribution². Given this, it is trivial to determine how self-interested players should vote in order to establish linear tax-rate T that would maximize their after-tax payoffs. The whole society of players essentially splits into two groups, which for convenience reasons we shall call 'rich' ($\frac{p_i}{\bar{p}} > 1 - C$) and 'poor' ($\frac{p_i}{\bar{p}} < 1 - C$). It is in direct interest of the rich to vote for zero-percent tax, while in the direct interest of the poor it is to opt for 100% redistribution. The 'intermediate class' has no interest at all in any concrete tax-rate. To avoid random voting, we may assume that all agents having maximized their own after-tax income, in the second place vote to maximize an average after-tax income of a community as a whole. This would lead 'intermediates' to vote in line with a non-redistribution principle self-interested voting scheme, in which we shall make a point of reference for further analysis, is presented in Figure 1.

It is clear that as the cost increases, the threshold value for relative initial income p_i/\bar{p} drops from 1 (for $C = 0$) to 0 (for $C = 1$). Thus for any given initial distribution of income a number of players championing a complete redistribution is a non-decreasing function of C . As the cost is approaching its absolute maximum at 1, the tax-rate T established by a popular vote in a society of self-interested players will be closer to 0. In an extreme case where $C = 1$, virtually no agent can gain from tax redistribution, and a laissez-faire system must prevail. Generally, we will refer to the tax-rate T established in a society of egoistic players as to the *Polarized Voting Tax (PVT)*³.

As opposed to taxes, redistribution through individual charitable transfers to other players involves no costs as there is no need to compel people to do what they are willing to do of their own initiative⁴. Thus if we

² This may be seen as an exemplification of positive/negative income tax, advocated by Milton and Rose Friedman (1996, pp. 114–119).

³ Similar redistribution mechanism to the one described above, though not allowing for voluntary charity transfers, was incorporated by Elizabeth Jean Wood (1999) into her model of rapid social change. Prof. Wood defined C to be an increasing function of T , and analyzed the model from a point of view of the decisive voter.

⁴ To be sure, it is simplifying assumption as private charity also incurs cost of collecting and distributing donations. However, to justify this feature of our model it is enough to notice that, as empirical evidence shows, the cost of private charity is substantially lower than in state-administered system (West, Ferris, 1999).

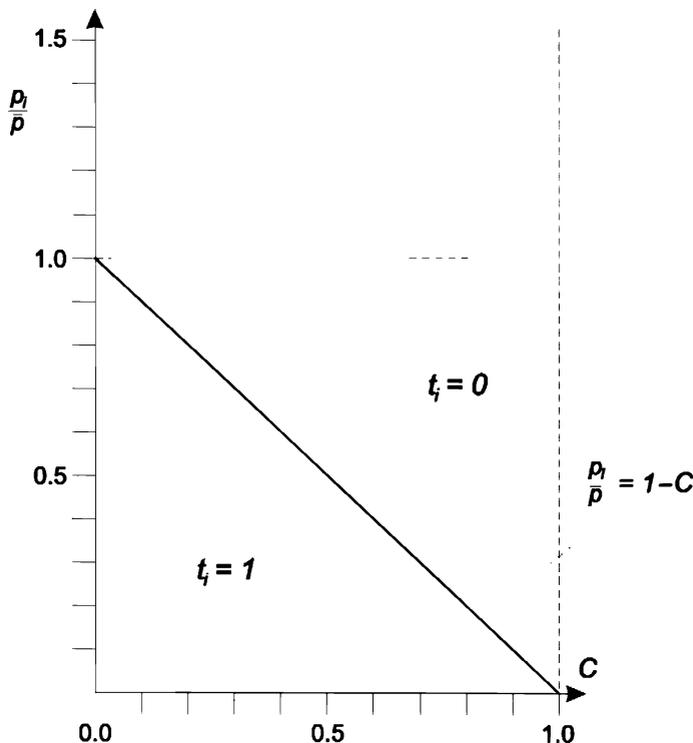


Figure 1. Polarized voting

denote charity transfer player i gives away as \hat{c}_i , and charity transfer player j receives as \check{c}_j , we can state that:

$$\sum_{i=1}^n \hat{c}_i = \sum_{j=1}^n \check{c}_j$$

In the following for simplicity reasons we will focus our attention on the redistribution process in a dyadic society with one rich and one poor player. However, it should be noted that this restriction obviously stops us from investigating some distinct new qualities of the game that emerge as number of agents exceeds 2.

Game with 2 players

Let us start with presenting a one-shot normal form redistribution game for two players. Suppose that players have different initial payoffs, and cost of execution satisfies following condition:

$$C < \frac{p_R - p_P}{p_R + p_P}$$

where p_P and p_R stand for poor and rich player's payoffs respectively.

The condition warrants that it is in the self-interest of the poor player to vote for 100% tax-rate which will result in establishing *Polarized Voting Tax (PVT)* equal to 50%. If cost of the execution exceeded the threshold value, even a poor player would incur losses at any positive tax level and therefore would be inclined to vote 0%.

It is obvious that at non-zero cost of the tax execution rich agent cannot opt for anything but 0% tax. Even if he is an altruist willing to support a poor player, it is better for him to do it by direct charitable payment which does not involve any cost. It is also plausible to assume that no poor agent will be interested in passing part of his initial payoff to the rich as this would further augment the original gap between himself and the latter. Therefore, charity transfer poor player gives away to the other in no case exceeds zero. Under these assumptions, rich votes 0% and chooses between alternative amounts of charitable transfer, while the poor one gives no donation and chooses how to vote between alternative tax-rates. Resulting from global tax-rate, as it is half the tax-rate proposed by the poor, varies between 0 and 50% (*PVT*).

With the help of the example suppose that initially a rich player earns \$25 and poor \$15, while the cost of execution is 10%. Suppose further that the rich considers giving \$2 to support the worse-off player, and the latter takes into account voting either 0 or 100%. Such a game can be presented in the following table:

Table 2

Redistribution game in normal form

		Poor	
		$t_P = 0 \rightarrow T = 0.0$	$t_P = 1 \rightarrow T = 0.5$
Rich	$\hat{c}_R = 2$	23.00 17.00	19.50 18.50
	$\hat{c}_R = 0$	25.00 15.00	21.50 16.50

Both people have dominant strategies: regardless of poor player's choice it is better for the rich player not to make any donation, and regardless of rich player's behavior it is more profitable for the poor to establish maximum 50% tax-rate by voting 100%. Thus, the equilibrium solution is for the rich

player not to contribute anything, and for the poor to vote for maximum redistribution possible (in which case the rich person's final payoff is \$21.50, whereas for the poor it is \$16.50). This, however, is not a Pareto-optimal result as for both players, it would be more profitable, had the rich chosen to donate \$2, and the poor decided to opt for a *laissez-faire* system (then the rich would have \$23 and the poor \$17). Thus, a redistribution game turns out to be an asymmetric variant of prisoner's dilemma.

The surplus players can divide between themselves the results from reduction of execution cost due to the virtual elimination of tax system. Holding players' initial payoffs constant, the lump-sum of the surplus depends on the actual tax-rate and the cost of execution.

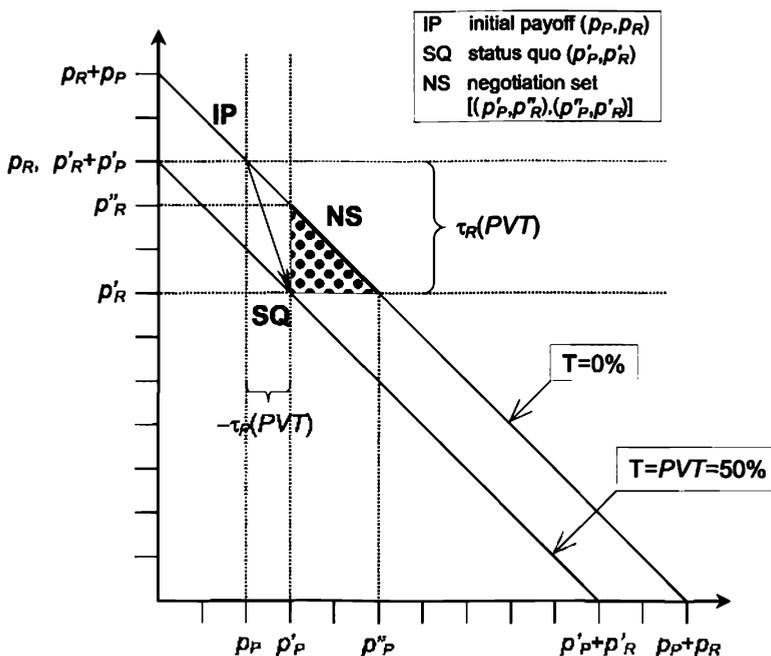


Figure 2. Negotiating surplus

In Figure 2 we see feasible outcomes of the game if players are in position to conclude a binding contract (poor and rich agent's payoffs at horizontal and vertical axes, respectively). They start the game with their initial payoffs (IP) when the rich earns p_R and the poor p_P . Under a democracy rule, a poor player can redistribute some wealth from the rich to himself by raising tax-rate up to 50% (PVT). By-product of tax redistribution is shifting of the budget constraint down and to the left. The shift occurs because at $T = 0.5$ some part of players' total payoff is consumed by the executive cost. Now

we may consider a point (p'_P, p'_R) to be status quo (SQ) since it is a pair of incomes that each agent is able to assure himself of his own, regardless of the other's behavior. At SQ the rich pays lump-sum tax equal to $\tau_R(PVT)$, and the poor receives a net subsidy equal to $-\tau_P(PVT)$. The difference between the two is the amount that could be re-gained by establishing laissez-faire system. However, the poor has no direct interest in lowering tax-rate for it would shift the outcome back in direction of IP , thus reducing his final payoff. On the other hand, 50% tax-rate brings harm to the rich player who not only covers the subsidy to the poor but also defrays the entire executive cost. Thus, it would be much to his interest to replace a politically-forced costly redistribution with a voluntary cost-free charity transfer to the other. To encourage the poor to vote for 0% tax-rate, though, the rich player needs to offer him a lump-sum at least equal to the loss incurred by the poor from eliminating tax redistribution, i.e. $-\tau_P(PVT)$. The thick black line in Figure 2 indicates the negotiation set, i.e. a number of solutions to the problem of how the surplus gained from abolition of tax system should be divided between the players. If entire surplus goes to the rich, the ultimate outcome will be (p'_P, p''_R) , if it falls solely to the poor, the outcome will be (p''_P, p'_R) . All combinations of payoffs between those two points are feasible as well.

Dynamical substitution between voluntary and forced redistribution

At this point we shall introduce dynamics into the system. Suppose the game is infinitely iterated, with initial payoffs and the cost of tax execution held constant and known to the players who have no possibility of direct communication. Each round will consist of the following sequence of moves: first players vote on redistribution, then tax transfers to and from the budget take place according to the current tax-rate, and finally it is up to players to give away some part of their income to the other.

At the outset poor player, willing to secure to himself the status quo outcome, votes for maximum redistribution and thus PVT at 50% is established. Now the rich player has an occasion to signal his willingness to replace tax redistribution with voluntary transfer by offering a donation to the poor. The lump-sum of this first donation depends on rich agent's *charitable initiative*. In the second round the poor player may react to the donation by reducing his demand for tax redistribution according to his personal *demanding attitude*. In turn the rich player reacts to

tax-rate decrease according to his *generosity*⁵, and so forth the process continues ad infinitum. Let us now formally define the individual features of players:

- ε – *charitable initiative*, is an amount of income, expressed as a share of $\tau_R(PVT)$, that a rich player is willing to give away directly to the other in the first round of the game; ε is effective as long as it satisfies the condition $\varepsilon \leq \frac{p_R - \tau_R(PVT)}{\tau_R(PVT)}$; greater values of ε are cut down to that threshold level, for donation cannot exceed rich agent’s entire income.
- γ – *generosity*, is a share of $\tau_R(PVT)$ that a rich player is willing to donate under laissez-faire system; effective γ ’s obey $\gamma \leq \frac{p_R}{\tau_R(PVT)}$.
- δ – *demanding attitude*, is a share of $-\tau_P(PVT)$ that a poor player will demand as a compensation for eliminating tax redistribution completely⁶.

Algorithm of decision making in each round of the game may be summarized in the table 3.

As we see in each round the poor player gives no donation ($\hat{c}_P = 0$), and the rich player votes for 0% tax-rate ($t_R = 0$). From the second round on tax-vote by the poor agent depends on his personal demanding attitude γ and the donation received in the preceding round. If at time $i - 1$ he received no charity transfer, at time i he will vote 100% (except for $\delta = 0$, in which case he votes 0% regardless of donation received); if transfer equaled $-\delta\tau_P(PVT)$ or more, he will be inclined to vote 0% (t_P ’s lower limit is obviously 0, so lower values are automatically increased to this level); if donation was somewhere between 0 and $-\delta\tau_P(PVT)$, he will depart from 100% vote proportionately. It is safety strategy for the poor to have $\delta \geq 1$. In case $\delta \geq 1$, he is ready to vote 0% only if rich agent fully covers the loss incurred by the poor from abandoning the status quo.

On the other hand, rich agent’s donation depends on his individual generosity γ and tax-rate T in the current round. If tax-rate is PVT , he

⁵ These three features (parameters) define a type of agent, though they are activated contextually, i.e. for a player in poor position only demanding attitude is relevant, whereas for a player in rich position it is charitable initiative and generosity. It should also be stressed that we use the terms in neutral sense and attach no moral value, neither positive nor negative, to any of the features. For instance, ‘generous’ actions may as well be motivated by strict self-interest.

⁶ Negative sign before $\tau_P(PVT)$ is due to the fact that at PVT poor player by definition ‘pays’ negative tax (which means that actually he receives net subsidy from the budget).

Table 3

Dynamics of redistribution game⁷

		Poor player	Rich player
Round no. 1	Voting	$t_P = 1$	$t_R = 0$
	Donation	$\hat{c}_P = 0$	$\hat{c}_R = \varepsilon \tau_R(PVT)$
Round no. 2 and next	Voting	$t_P = \begin{cases} 0 & \delta=0 \\ 1 + \frac{\check{c}_P}{\delta \tau_P(PVT)} & \delta>0 \end{cases}$ * $\check{c}_P = \hat{c}_R$ of the preceding round	$t_R = 0$
	Donation	$\hat{c}_P = 0$	$\hat{c}_R = \begin{cases} 0 & PVT=0 \\ \frac{PVT-T}{PVT} \cdot \gamma \tau_R(PVT) & PVT>0 \end{cases}$ * T of the current round

gives no donation, regardless of his generosity; if T is 0%, his donation equals $\gamma \tau_R(PVT)$; if T is somewhere between 0 and PVT , he offers a proportionate charity transfer. His safety strategy is to have $\gamma \leq 1$. In case $\gamma = 1$, he is ready to donate to the poor entire surplus resulting from abolishing tax redistribution. Was his generosity greater than that, he would expose himself to the risk of earning less than status quo.

Under what conditions can tax redistribution be completely replaced with voluntary transfers on part of the rich? To answer this, let us find levels of ε , γ , and δ that make it possible to establish laissez-faire system. Roughly speaking, to make a tax-rate go down to zero, it is necessary that the rich was generous *enough*, whereas the poor was not demanding *too much*. A threshold value of poor's demanding attitude (δ) as a function of rich's generosity (γ) is given by the *correspondence formula*⁸ (see Appendix for details):

$$\delta = -\frac{\tau_R}{\tau_P} \cdot \gamma$$

⁷ The same algorithm may be applied to multiplayer game. Tax-vote by poor player depends on the donation he received in the previous round, though when number of players exceeds two, the donation need not be equal to the transfer made by any particular rich agent (one possible way to distribute charity transfers among the poor is to employ leximin principle). The 'intermediate class' could be defined to have $t_I = 0$ and $\hat{c}_I = 0$ for each round.

⁸ The visual presentation of the equation will be referred to as *correspondence line*. To avoid excessive notation, from now on τ_R and τ_P will denote lump-sum tax paid at PVT rate.

If δ satisfies the equation than substitution rate between taxes and free donations is exactly the same for both agents. It means that in order to completely eliminate taxes the rich player is ready to give away the amount that is precisely as much as poor player demands for reducing his tax-vote to zero. If δ exceeds the threshold value, tax-rate T will be equal to PVT from the very beginning, or will be approaching limit at PVT with speed negatively correlated to the charitable initiative of the rich (ε). If δ is less than that, at some point of the game laissez-faire system will be established (the smaller δ and the larger ε , the sooner tax-rate will fall to zero). If δ is exactly equal to the threshold value, than the tax-rate established in the second round will hold for the rest of the game. Thus ultimate tax-rate in this case is determined by ε and may assume any value between 0 and 1. Its exact value is given by (see Appendix):

$$T = \frac{1}{2} \left(1 - \frac{\varepsilon}{\gamma} \right), \quad \gamma > 0, \varepsilon \leq \gamma$$

Payoff structure at time approaching infinity

To conclude the analysis we will examine limit payoffs, as time approaches infinity, for different combinations of generosity γ and demanding attitude δ , while holding charitable initiative ε equal to γ ⁹. Next we will point pairs of γ and δ that form Nash equilibria. To make the analysis easier to follow we will plot the results on the same diagram as seen in Figure 3.

Let us denote the rich agent's status quo payoff (PVT , no donation) as R_0 , payoff better than that as R_+ , and worse one as R_- ; and respectively for poor agent: P_0 , P_+ , P_- . In Figure 4 we show how given combinations of γ and δ prove better/worse than status quo for either player.

If generosity γ of the rich agent is too small in comparison to demanding attitude δ of the poor, the tax-rate will approach a limit at PVT , and thus both players will receive their status quo payoffs R_0 , P_0 (entire area above correspondence line). However, if γ and δ are kept in 'reasonable' proportion to each other, a lower tax-rate is established, and there is a surplus resulting

⁹ For a rich agent playing his safety strategy ($\gamma \leq 1$), it is always profitable to have laissez-faire system established. As we focus on the limit payoffs at infinity, single first round charity expense determined by ε does not affect rich agent's payoff, so he does not incur any more risk by giving ε value equal to (or even higher than) δ . And by doing so he is able to reduce tax-rate to zero in case γ and δ happen to lie on the correspondence line.

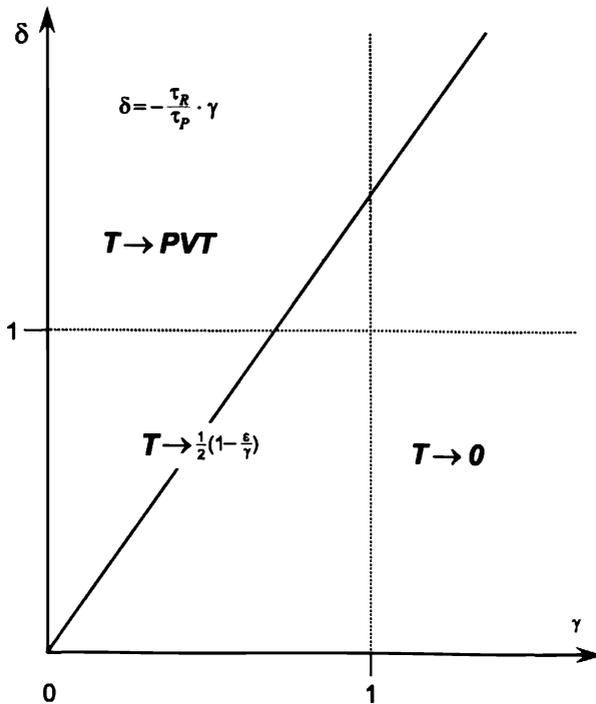


Figure 3. Generosity, demanding attitude and tax-rate

from cutting down on execution costs. Thus on and below the correspondence line always at least one of the agents is better-off than in status quo. It's worth noticing that for any given value of γ both agents' limit payoffs are independent of δ , as far as δ does not exceed the threshold value¹⁰. Particularly, for $\gamma = -\tau_P/\tau_R$ entire surplus is taken by the rich, while the poor is left with his status quo earning, and for $\gamma = 1$ entire surplus goes to the poor, while the rich keeps his status quo income. Generally, for all points on and below the correspondence line, as γ coordinate is reduced, the rich agent's payoff increases 'at the expense' of the poor.

We may now see that when considering limit payoffs redistribution game is basically a variant of ultimatum game. The rich agent proposes a donation depending on his generosity γ , and the poor agent either accepts the offer or rejects it depending on his demanding attitude δ . In case δ lies above a correspondence line and the proposition is rejected, both players are left with their status quo payoffs. In contrast to the original ultimatum game

¹⁰ The reason for it is that parameter γ by its very definition determines the lump-sum that goes to the poor by means of voluntary transfer under laissez-faire system, and as we know from Figure 3 the limit tax-rate under correspondence line is zero.

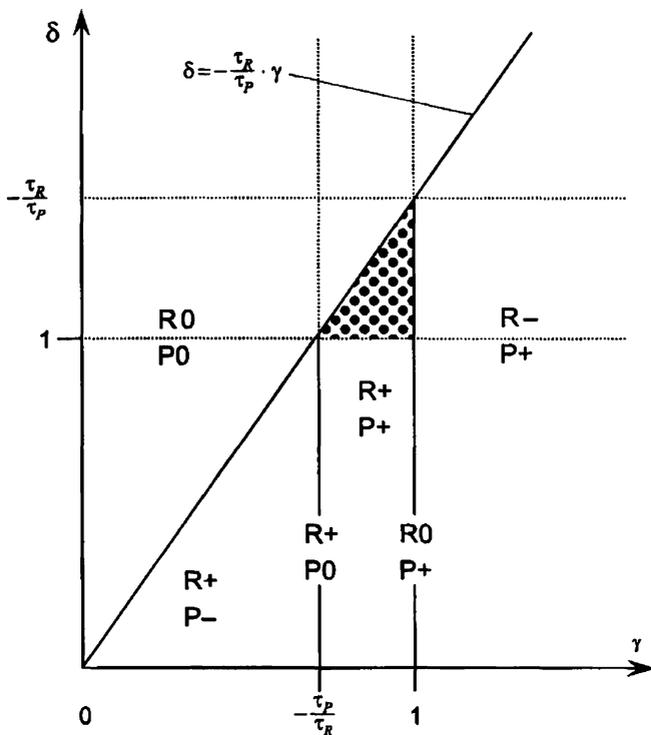


Figure 4. Limit payoffs in comparison to status quo

where players' payoffs could not fall below zero, reaching an 'agreement' in the redistribution game may lead to one of the players towards being materially worse-off than in status quo. The latter situation is possible when either player does not play his safety strategy.

As we may read from the diagram, safety strategy for the poor player is to have $\delta \geq 1$. Was δ below 1, the poor could suffer loss in comparison to status quo, if the rich had his γ below $-\tau_P/\tau_R$. Similarly it is safe for the rich to have $\gamma \leq 1$. Was his generosity greater than that, his limit income could fall short of status quo, in case the poor was not too demanding ($\delta \leq -\tau_R/\tau_P$). The dotted triangle indicates pairs of safe γ and δ that bring profit to both sides (except for point $(-\tau_P/\tau_R, 1)$ and the right side of the triangle, where only one player gains, while the other stays with status quo payoff). All γ - δ pairs in the triangle (and rectangle below it as well) lead to payoffs that belong to the negotiation set presented in Figure 2.

Further scrutiny leads us to the conclusion that for a self-interest maximizing poor player $\delta = 1$ is a dominant strategy. Such a choice is the analogue of the receiver accepting zero-share in ultimatum game with continuous payoffs.

Nash equilibria of the redistribution game

Finally, let us point out combinations of γ and δ that form Nash equilibria of the redistribution game. First of all, we may rule out all points that violate either agent's safety level, i.e. $\gamma > 1$ or $\delta < 1$. Further let us consider γ belonging to $(-\tau_P/\tau_R, 1]$. The poor player's best response to any given γ in the range is **any** δ less or equal to threshold value. However, rich agent's best response for any δ less or equal to $-\tau_R/\tau_P$ is choosing γ in such a way as to locate the point (γ, δ) on the correspondence line. Thus all points on a hypotenuse of a dotted triangle in Figure 4 are Nash equilibria (thick line in Figure 5), while the others are not¹¹.

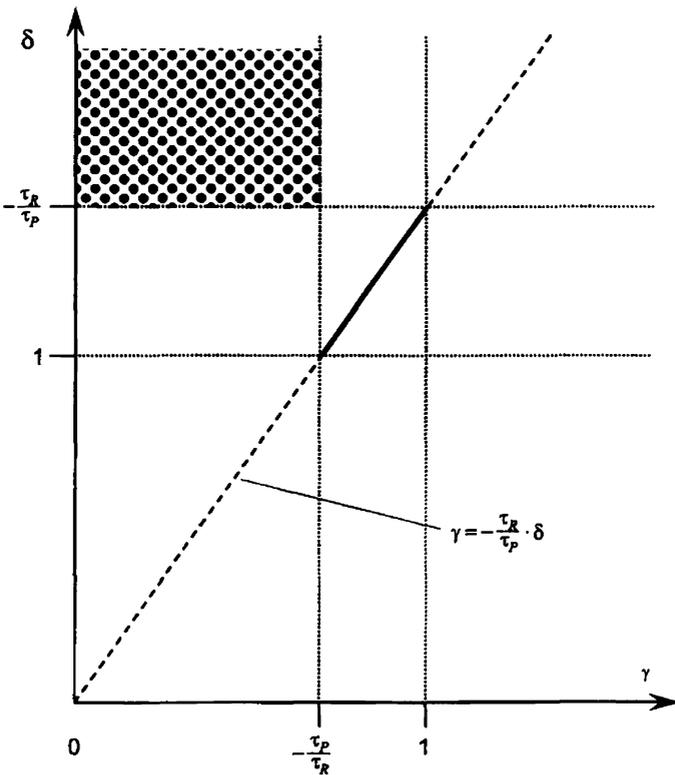


Figure 5. Nash equilibria (for $\varepsilon = \delta$)

¹¹ As $\delta = 1$ is a poor player's dominant strategy, combination of $\delta = 1$ and $\gamma = \varepsilon = -\tau_P/\tau_R$ is a unique solution of the redistribution game for strictly self-interested players. In this case poor player gets exactly his status quo income while the rich increases his status quo payoff by re-gaining the entire surplus resulting from abolition of executive costs of tax system.

For γ belonging to $[0, -\tau_P/\tau_R]$ any level of δ over the correspondence line may be considered the best response, as well as for δ equal to or greater than $-\tau_R/\tau_P$ the best response could be any γ to the left of the correspondence line. Thus Nash equilibria are also all points of the dotted rectangle, although they obviously are not Pareto-optimal. As empirical evidence from dictator and gangster games shows, it may well be the case that subjects actions lead to suboptimal outcomes. "The generous nature of individuals found in fairness games does not overcome the distribution struggle. While dictators are prepared to give up a part of their endowment, gangsters demand a much bigger share of the cake for themselves" (Eichenberger Oberholzer-Gee 1998, p. 196).

Graphical illustration of system dynamics

To give an example of redistributive dynamics we will conclude the article with a few characteristic cases of system evolution. In each case agents' initial payoffs are \$90 (rich) and \$10 (poor), whereas the cost of tax execution is 20%. At *PVT* (50%) the rich agent's payoff is equal to \$65 and the poor player earns \$25. On each graph a line - - - refers to the rich agent's final income (all taxes and voluntary transfers included), a line -.-.- to the poor agent's income, and a line -.-.- reports the history of the tax-rate.

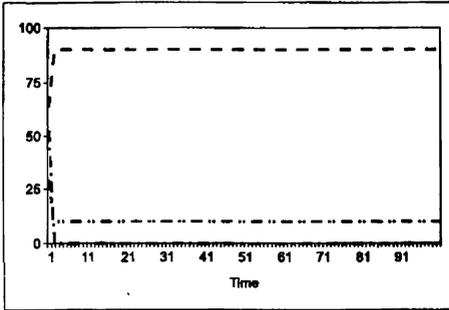
Illustration no. 1 shows how a stable state is immediately reached if the poor agent exhibits no demand for redistribution ($\delta = 0$). If at the same time the rich agent exhibits no generosity ($\gamma = 0$), he takes entire surplus resulting from eliminating executive costs, and both players earn their initial payoffs. It may be seen as an instance of the poor player's high moral standards that hold him back from exploiting the democratic procedure to enforce more profitable income distribution. To be sure, that sort of consideration would not even pass through the mind of *homo oeconomicus*.

Illustration no. 2 shows that if the poor agent's demanding attitude is anything apart from zero, it is impossible to establish laissez-faire system without charitable initiative of the rich ($\varepsilon = 0$) – no matter how great his generosity could be ($\gamma \geq 0$). The tax-rate does not deviate from *PVT* even by the smallest margin and in each round the agents earn their status quo payoffs (\$65 and \$25).

Graphs nos. 3 and 4 show the history of reaching Pareto-optimal distribution (with both agents' payoffs higher than status quo). In no. 3 the rich player does not play his best response. By reducing his generosity and

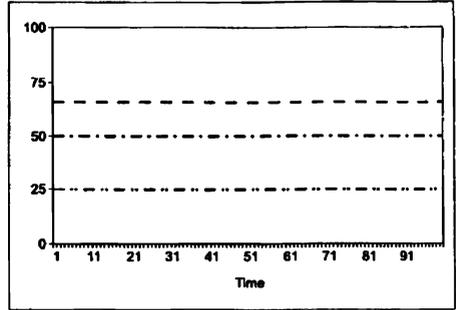
1. No demand for redistribution

$$\gamma=0.00 \quad \varepsilon=0.00 \quad \delta=0.00$$



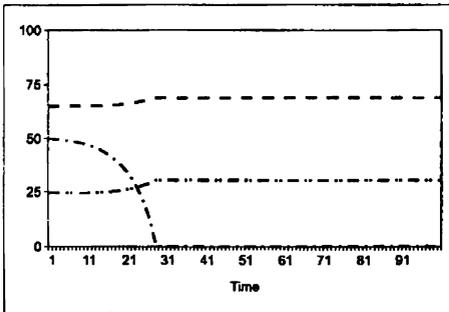
2. No charitable initiative (suboptimal NE)

$$\gamma \geq 0.00 \quad \varepsilon=0.00 \quad \delta > 0.00$$



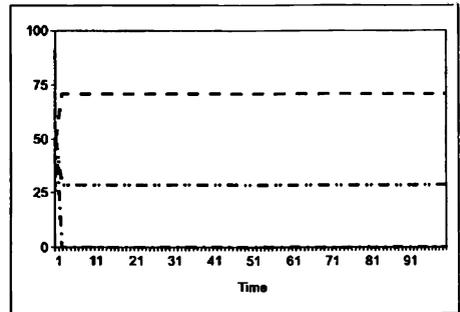
3. Reaching Pareto-optimal outcome (not NE)

$$\gamma=0.85 \quad \varepsilon=0.01 \quad \delta=1.25$$



4. Pareto-optimal Nash Equilibrium

$$\gamma=0.75 \quad \varepsilon=0.75 \quad \delta=1.25$$

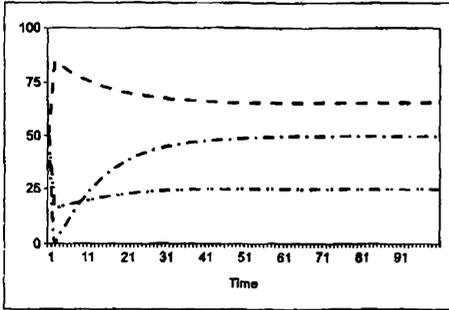


raising his charitable initiative appropriately, he could assure himself a higher payoff, taking benefit of the poor agent's moderate demand for redistribution. If he did so, Nash equilibrium presented in graph no. 4 would be established: $(\gamma = 0.75, \delta = 1.25)$ is a point lying exactly on the correspondence line.

Graphs nos. 5 and 6 show how system can recede to *PVT* after initial reduction of tax-rate to nigh-zero level. No. 5 illustrates that it is impossible to dupe the poor player into *laissez-faire* system by substantial charitable initiative combined with low generosity. Even though the poor agent plays δ far below his safety strategy, the tax-rate after the initial reduction gradually recedes to *PVT*. On the other hand, in no. 6 poor player's demand for redistribution is too much even for a super-generous rich agent. In spite of large initial charity transfers (that even made the recipient wealthier

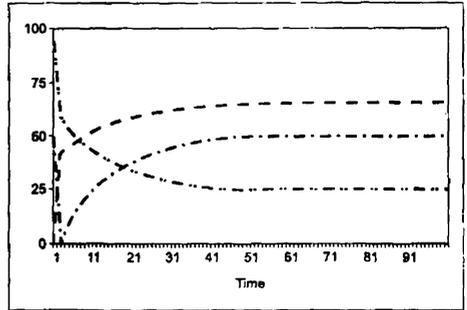
5. Recession to PVT due to insufficient generosity

$\gamma=0.20 \quad \varepsilon=0.40 \quad \delta=0.36$



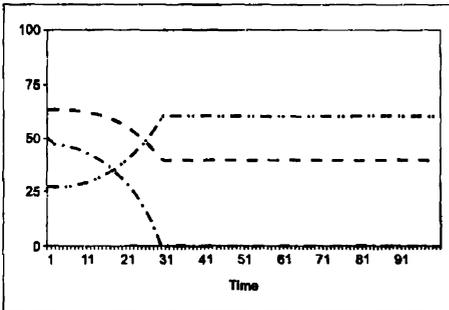
6. Recession to PVT due to excessive demand

$\gamma=2.00 \quad \varepsilon=2.50 \quad \delta=3.60$



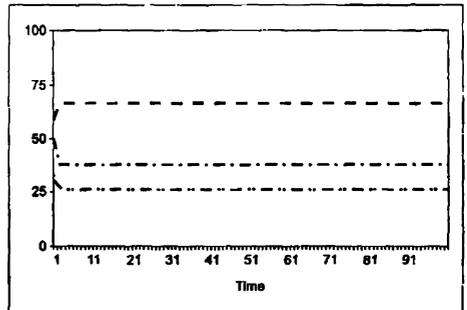
7. Super-generosity

$\gamma=2.00 \quad \varepsilon=0.10 \quad \delta=3.00$



8. Insufficient charitable initiative

$\gamma=0.80 \quad \varepsilon=0.20 \quad \delta=1.33$



than his benefactor), the poor player departs from voting 0% and *PVT* is gradually brought back.

Illustration no. 7 depicts a rich man who is willing to give away most of his income, starting with a small initial contribution. The poor agent's demanding attitude is below the threshold value, so the tax-rate is being gradually reduced to zero. The poor agent takes benefit of the rich agent's generosity, receiving a substantial part of his wealth, and ultimately becoming richer than the donor.

Finally, at no. 8 we see a game between two players whose generosity and demanding attitude are exactly corresponding to each other. However, due to insufficient charitable initiative on the part of the rich, the tax-rate is fixed at 37.5% and benefits from complete eliminating executive costs are lost.

Concluding remarks

The purpose of the article was to model a redistribution process, allowing for the interplay between transfers forced by means of the tax system and voluntary donations to the worse-off. The dynamics of the system were guided by agents' personal features, namely charitable initiative and generosity on part of the rich, and demanding attitude on part of the poor. We have shown that even under the assumption of exclusive self-interest seeking, there are Pareto-optimal Nash equilibria that result in a complete substitution of free charity for tax redistribution, as well as suboptimal equilibria that keep the volume of tax redistribution intact. It is a question of empirical research whether real-life subjects are able to find their way to elimination of excessive cost of politically forced transfers. It is also a matter for further discussion how dynamics of the game are affected by introducing greater number of players and focusing attention on discounted payoffs rather than looking at the limit distribution of income.

Appendix

To simplify notation let the variables over time be denoted as:

- c_i – voluntary transfer from rich to poor player in round i ,
- t_i – poor agent's tax-vote in round i ,
- T_i – tax-rate in round i ,

Constants (as described in the text):

$$p_R, p_P, \bar{p}, \tau_R, \tau_P, C, \varepsilon, \gamma, \delta$$

At time i player k 's payoff is given by formula:

$$(1) \quad p_k(T_i) = (1 - T_i)p_k + T_i(1 - C)\bar{p}$$

According to decision algorithm (see Table 3), rich agent's donation in first round is given by:

$$(2) \quad c_1 = \varepsilon\tau_R$$

His donations in round $i \geq 2$ are:

$$c_i = \frac{PVT - T_i}{PVT} \gamma\tau_R.$$

Since $PVT = \frac{1}{2}$, we get

$$(3) \quad c_i = (1 - 2T_i)\gamma\tau_R.$$

Tax-rate at time i is always half the tax-rate proposed by the poor (again see Table 3):

$$(4) \quad T_i = \frac{1}{2}t_i = \frac{1}{2} \left(1 + \frac{c_{i-1}}{\delta\tau_P} \right)$$

Substituting (2) into (4), we obtain tax-rate in second round:

$$T_2 = \frac{1}{2} \left(1 + \frac{\varepsilon\tau_R}{\delta\tau_P} \right).$$

Substituting (3) into (4), we obtain difference equation for $i \geq 3$:

$$T_i = -\frac{\gamma\tau_R}{\delta\tau_P}T_{i-1} + \frac{1}{2} \left(1 + \frac{\gamma\tau_R}{\delta\tau_P} \right).$$

To make it easier to handle let us rewrite it as:

$$(6) \quad T_i = aT_{i-1} + b,$$

where $a = -(\gamma\tau_R/\delta\tau_P)$, $b = \frac{1}{2}(1 - a)$.

Or, alternatively:

$$(7) \quad T_i = a(T_{i-1} - \frac{1}{2}) + \frac{1}{2}.$$

Now the sequence of T_i 's for $i \geq 3$ is:

$$T_3 = aT_2 + b$$

$$T_4 = aT_3 + b = a^2T_2 + ab + b$$

\vdots

$$T_i = a^{i-2}T_2 + a^{i-3}b + a^{i-4}b + \dots + ab + b = a^{i-2}T_2 + \frac{1-a^{i-2}}{1-a}b = a^{i-2}T_2 + \frac{1}{2}(1 - a^{i-2}) = a^{i-2} \left(T_2 - \frac{1}{2} \right) + \frac{1}{2}$$

Thus for $i \geq 3$:

$$(8) \quad T_i = a^{i-2} \left(T_2 - \frac{1}{2} \right) + \frac{1}{2}.$$

Solving (8) with $i \rightarrow \infty$ we get:

1. $0 \leq a < 1$:

$$a^{i-2} \rightarrow 0 \text{ and } \lim_{i \rightarrow \infty} T_i = \frac{1}{2}$$

2. $a = 1$:

$$T_i = T_2 \quad \forall i$$

3. $a > 1$:

if $T_2 = \frac{1}{2}$ (which implies $\varepsilon = 0$ and $\delta > 0$): $T_i = \frac{1}{2} \quad \forall i$,

if $0 \leq T_2 < \frac{1}{2}$: $\exists m \geq 2$ for which $T_m \leq 0$ (since in (8) the expression in parentheses is negative, and a^{m-2} increases in m). Since any T lower than zero is automatically increased to zero, we have $T_m = 0$.

Now let m be the smallest possible number of a round. In accordance with (7), if $T_m = 0$, then $T_{m+1} = (1 - a)/2 < 0$. Therefore $T_{m+1} = 0$, and since the same holds true for all subsequent rounds, $\lim_{i \rightarrow \infty} T_i = 0$.

Deciphering a , we get:

1. For $\delta > -\frac{\tau_R}{\tau_P} \cdot \gamma$, $\lim_{i \rightarrow \infty} T_i = \frac{1}{2}$, $\gamma > 0$. If $\gamma = 0$, $T_i = 0 \forall i \geq 2$ (see Table 3).
2. For $\delta = -\frac{\tau_R}{\tau_P} \cdot \gamma$, $\lim_{i \rightarrow \infty} T_i = T_2 = \frac{1}{2} \left(1 + \frac{\varepsilon \tau_R}{\delta \tau_P} \right)$, $\delta > 0$. Since $\delta = -\frac{\tau_R}{\tau_P} \cdot \gamma$, after transformation we receive: $\lim_{i \rightarrow \infty} T_i = \frac{1}{2} \left(1 - \frac{\varepsilon}{\gamma} \right)$, $\gamma > 0$. If $\delta = \gamma = 0$, $T_i = 0 \forall i \geq 2$ (see Table 3).
3. For $\delta < -\frac{\tau_R}{\tau_P} \cdot \gamma$,
 - a) if $\varepsilon = 0$ and $\delta > 0$, $\lim_{i \rightarrow \infty} T_i = \frac{1}{2}$;
 - b) if $\varepsilon > 0$, $\lim_{i \rightarrow \infty} T_i = 0$.

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