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ORIGINAL PAPER

FORMING OF THE DYNAMICS OF THE CHANGES IN CONVERGENT PRODUCTION SYSTEM DEPENDING ON SIZE OF PRODUCTION PARTY

Bożena Zwolińska[1], Łukasz Kubica[2]

1) AGH University of Science and Technology, Kraków, **Poland** 2) Jagiellonian University in Kraków, Kraków, **Poland**

ABSTRACT. **Background:** In terms of Lean Six Sigma, the whole process focuses on clients and their needs. Existence of a client generates the supply of companies. Extended customization has a negative impact for a structure of the production system. Dynamics of changes and no predictability of system's state in time t+1 lead to increase of the operational costs. It particularly affects those companies which are producing goods using MTO (make – to – order) method in short series. The goal of this article is to establish a mathematical model defining how the structure of a production system is subject to change depending on the volume of the production batch for a production system in accordance with MTO. Furthermore pilot calculations have been presented which determine the probability value, how subsequent random variables are contained within three standard deviations (±3δ) from the determined expected value (ET) for the entire production structure. Months of analysis and research on introducing selected lean toolbox components to a polish company from the small and medium enterprises sector resulted in the models presented in the article. The production structure of the discussed actual facility is complex and is of converged nature in accordance with MTO, while the final products are manufactured in short production series with a relatively wide customization options.

Materials and results: Wrought models consider theories of Klir and Maserovicz [Mesarovic 1964] and also theory of mass operation (one of the probability areas). In the article there are results from two models which are fundamental in defining problems in logistics engineering and production in scientific research. Important attribute of presented models is a fact that they consider relations between variables in a structure of consecutive processes and also consider relations between a size of production party and a real object.

Presented models are not only theoretical coverage but also consider real relations between objects. Real productive object specialized in producing cooling devices destined to store hematogenous objects, plasma and cryoprecipitate has been analyzed. Those devices have very strict quality requirements (consistent with ISO 13485 and CE0434 in accordance with Directive 93/42/EEC).

In the article there is a presentation of three models which indicates two different functions of production time for production party of $2 \leq k \leq 30$ and $k > 30$ items. In the following model there are a few different parameters of the production system: variable parameters of processes' times which depend on a kind of half-finished product, dependency of time needed to produce an item, size of a production party and also dependency of operational times and implemented technology.

Conclusions: It is important to customize tools to individual attributes of a system whilst implementing changes in real objects. One change can be effective in one organization and not necessarily in the other. Wrought model is a first of the steps in building a scheme necessary to validate a real object in time t+1. On the next step those theories will be implemented in IT tool environment of R Studio or Witness System Simulation Modeling to conduct statistical analysis based on historical data.

Key words: Convergent production system, model of the changes' dynamics

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INTRODUCTION

Thin management of the company assumes no failures of devices and no resources in the whole chain of deliveries [Wiegand et al. 2005] and also minimization of possible 3M losses (Jap. „muri", „mura" and „muda"). Good examples are automotive companies in which implementation of lean toolbox ends up with a success and effects are visible and also are reliable in relatively short period of time. In a way worse situation are Polish companies from the group of "small and medium companies". In those companies investments for trainings and growing of Kaizen consciousness are lower. It causes failure in implementing of Lean philosophy. Lean methodology contains adjustments in tasks of delivery chain in three different aspects: CLIENT, PROCESS, EMPLOYEE [Michlowicz et al. 2015]. In terms of Lean Six Sigma, the whole process focuses on clients and their needs [Nyhuis and Windhal 2009]. Existence of a client generates the supply of companies. Existence of a client generates the supply of companies. In production without a need for final product there are some 7M wastes generated (7 "muda"). Overproduction is considered to be the worst loss, because it generates consecutive wastes as: unnecessary usage of material, additional resources, unnecessary movements etc. [Zwolińska 2016].

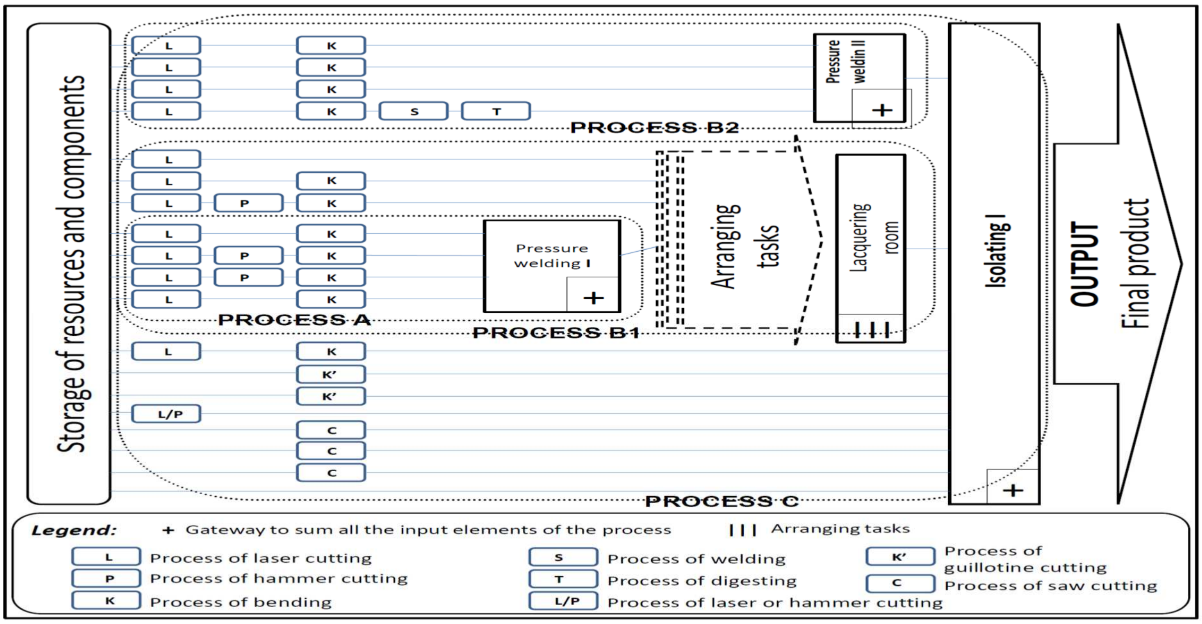
Production strategies can be divided into two elementary groups: MTS (make – to – stock) and MTO (make – to – order). Whilst implementing one of those strategies, it is important to analyzed benefits and potential risks/losses of this implementation. Companies which produce within MTO method have to be elastic and be able to adjust to dynamic changes of environment (as clients' needs). Extended customization has a negative impact for a structure of the production system. Dynamics of changes and no predictability of system's state in time t+1 lead to increase of the operational costs [Koliński et al. 2016]. It particularly affects those companies which are producing goods using MTO method in short series (up to 20 pieces of the same product).

Real productive object specialized in producing cooling devices destined to store hematogenous objects, plasma and cryoprecipitate has been analyzed. Those devices have very strict quality requirements (consistent with ISO 13485 and CE0434 in accordance with Directive 93/42/EEC).

Main purpose of this article is to formulate model and counting algorithm of convergent production system. Basic parameter of rating is variability of analyzed model depending on size of production party. Researched model is not a deterministic system that is why in consideration variability is considered as variation in comparison to expected value in certain random variable. All considered times of cycles are random variables for which there were assigned certain thickness probabilities with optimal parameters (in previous research). In the article there is a presentation of results of two different variants of production party: $2 \leq k \leq 30$ and $k > 30$; where k is random variable in Poisson composition and it sets amount of stocks of the same assortment which results from orders made in Δt time.

DEFINING OF THE ANALIZED PRODUCTION SYSTEM

Considered structure has convergent character. It means that as results of a few degrees of processing of n elements and input resources – final products is made. Moreover, analyzed model is a system made of a few hierarchic sub-processes. First phase of analysis covered designation of complex production structure for system of dependent sub-systems. Decomposition for smaller sub-systems is valid and compatible with general theory of complex systems [Buslenko et al. 1979; Gniedenko and Kowalenko 1966, Klir 1976; Mesarovic 1964]. Figure 1 presents scheme of analyzed system in process after decomposition and classification for certain sub-systems.



Source: own elaboration

Fig. 1. Scheme of analyzed structure in process
Rys. 1. Schemat w ujęciu procesowym analizowanej struktury

Analyzed production system *SP* is defined as a set:

$$SP = \{E, A, X, Y, R\} \tag{1}$$

where: E – set of SP stocks; A – stocks' attributes SP; X – parameters of SP input; Y – parameters of SP output; R – relations between: E, A, X and Y in the SP area. Moreover:

$$E = \left\{ \left\{E_1^1, E_1^2, ..., E_1^{N_1}\right\}; \left\{E_2^1, E_2^2, ..., E_2^{N_2}\right\}; ...; \left\{E_N^1, E_N^2, ..., E_N^{N_N}\right\} \right\} \tag{2}$$

where: $E_1^{i_1}$ – elements of the 1 set (machine) which has been assigned in according to the executive technology process, for example cutting – set N_1 – amount of cutting machines,; $E_2^{i_2}$ – elements of the 2 for example.: N_2 – amount of the edge press.

$$A = \left\{ \begin{matrix} \left(a_1 \sim E_1^{N_1}\right) | \; y_{M,N}^K ; \left(a_2 \sim E_2^{N_2}\right) | \; y_{M,N}^K ; ... ; \\ \left(a_n \sim E_n^{N_n}\right) | \; y_{M,N}^K \end{matrix} \right\} \tag{3}$$

where: a_n – attribute dependent on $E_n^{N_n}$ whilst using the element (or set of elements) from $y_{M,N}^K$.

$$X = \left\{ D^K, K \,|\, D^K, D^S, S \,|\, D^S, D^P, P \,|\, D^P, R^X \right\} \tag{4}$$

where: D^K – suppliers of the input components; K – input components dependant on D^K; D^S – suppliers of the input stocks; S – input stocks dependant on D^S; D^P – suppliers of the Energy; P – stream of Energy dependant on D^P; R^X – relations between X set.

$$Y = \left[\begin{array}{cccc|cccc|c|cccc} y_{1,1}^1 & y_{1,2}^1 & \cdots & y_{1,N}^1 & y_{1,1}^2 & y_{1,2}^2 & \cdots & y_{1,N}^2 & \cdots\cdots & y_{1,1}^L & y_{1,2}^L & \cdots & y_{1,N}^L \\ y_{2,1}^1 & y_{2,2}^1 & \cdots & y_{2,N}^1 & y_{2,1}^2 & y_{2,2}^2 & \cdots & y_{2,N}^2 & \cdots\cdots & y_{2,1}^L & y_{2,2}^L & \cdots & y_{2,N}^L \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\cdots & \cdots & \cdots & \cdots & \cdots \\ y_{M,1}^1 & y_{M,2}^1 & \cdots & y_{M,N}^1 & y_{M,1}^2 & y_{M,2}^2 & \cdots & y_{M,N}^2 & \cdots\cdots & y_{M,1}^L & y_{M,2}^L & \cdots & y_{M,N}^L \end{array} \right] \tag{5}$$

where: $y_{M,N}^L$ – single final product for which K, M, and N indexes are: different size, different models and destination and also variety of the additional options. Moreover, $y_{M,N}^L$ is a single column matrix which consists of the elements of the Bill of Materials structure.

In considered system, there are m single production streams – due to analytical subsystem disintegration. Single production stream is signed with l, where $l = 1, 2, \ldots, m$. In each stream there are n processes to be executed. It is signed with i for single process for $i = 1, 2, \ldots, n$. In each process there are k elements (tasks). Number of necessary tasks is signed by j for $j = 1, 2, \ldots, k$. For *SP* production system there were random variables set to characterize realization times for each step of the production process. Let's consider case of k elements: first case for $2 \le k \le 30$ stocks (for example: 2, 5 or 10 stocks); second example $k > 30$ stocks (for example 100 stocks). In fact k is a random variable of Poisson composition and it sets the amount of stocks of the same product which is caused by the order which took place in time Δt. In considered models we will use an example in which k is deterministic.

We sign $\tau_{l,i,j}$ as partial time to realize one task in l – stream on i – process (machine) for j – element. After analyzing historical data and analysis we know that $\tau_{l,i,j}$ has exponential composition with optimal parameter of $\lambda_{l,i}$ – the same for each j – element (task) in l – stream for i – process, – so $\tau_{l,i,j} \sim \text{Exp}(\lambda_{l,i})$. Moreover, we know from analysis that each final product is composed with 160 – 180 different objects. Approximately 35 – 40 % of those are composed by the components ordered externally, which are not processed in *SP*. The rest of the elements are semi-products created in a factory – approximately 105 – 120 different stocks from BOM – *Bill of Materials* structure. Each semi-product has its' own (individual) transition path. The summary of realization times of one product is a sum of times ($\tau_{l,i,j}$) of each step (production process) in l – stream. In our particular production stream there are n different processes, that is why we can assume that $\lambda_{l,x} \neq \lambda_{l,y}$ for each $x, y \in \{1, 2, \ldots, i\}$ and $x \neq y$. That is why function of thickness for random variable T_l which describes whole time of executing n processes for each product can be described with formula [Singh and Dattatreya 2007]:

$$f(\tau) = \left[\prod_{i=1}^{n} \lambda_{l,i}\right] \cdot \sum_{h=1}^{n} \frac{e^{-\lambda_{l,h} \cdot \tau}}{\prod_{\substack{g \neq h \\ g=1}}^{n} \left(\lambda_{l,g} - \lambda_{l,h}\right)} \tag{7}$$

where: $\lambda_{l,i}$ – parameter of exponential composition depends on l-stream and i-process.

To set what will be expected time of realization of all necessary tasks for single final product we should assign expected value of variable T_l for each $l = 1, 2, \ldots, m$ accordingly to the formula:

$$ET_l = \sum_{i=1}^{n} \frac{1}{\lambda_{l,i}} \tag{8}$$

Thanks to the fact that random variables T_l for each $l = 1, 2, \ldots, m$ are independent – expected value of random variable T (which describes the whole realization time) can be calculated this way:

$$ET = ET_1 + ET_2 + \ldots + ET_m \tag{9}$$

We know (from historical data) that a need of analyzed group of products in particular months varies from 60 to 80 stocks and a single production party was about 5 to 12 stocks of identical products. To minimize losses, production schedules adjusted tasks for cumulated orders. That is why, in the next part of the article there will be analysis of production party where k will be $2 \le k \le 30$.

Shaping of a realization time of order for $2 \le k \le 30$

If $\tau_{l,i,j}$ – is a random variable which sets single realization time of j – task in single i – production process for l – stream, then a sum of time for realization of k elements in i – process in j-stream can be set by using a formula:

$$t_{l,i} = \tau_{l,i,1} + \tau_{l,i,2} + \ldots + \tau_{l,i,k} \tag{10}$$

where: $t_{l,i}$ – random variable which describes the whole realization time of k necessary objects (tasks) in i – production process (step) for l – value stream. Moreover, we know that $\tau_{l,i,j} \sim \text{Exp}(\lambda_{l,i})$, for all $j = 1,2 \ldots k$, so $t_{l,i} \sim Erl(k, \lambda_{l,i})$.

As $t_{l,i}$ is a sum of k independent random variable $\tau_{l,i,j}$ with the same exponential composition with a parameter $\lambda_{l,i}$, characteristic for a single process and single part – we can assume, that $\lambda_{l,i}$ have different pairs. If there are $x, y: \lambda_{l,x} = \lambda_{l,y}$, then $t_{l,x}+t_{l,y} \sim Erl\,(x+y, \lambda_{l,x})$ and we can use following procedure.

We have to assign random variable with T_l - it will set whole time of k elements passing through l – stream, then:

$$T_l = t_{l,1} + t_{l,2} + t_{l,3} + \ldots + t_{l,n} \tag{11}$$

where: T_l – random variable describing the whole time of processes realization for l-stream of value; $t_{l,1}, t_{l,2}, \ldots, t_{l,n}$ – are random variables which set the realization time for k elements (tasks) on i – process in l – stream.

Basing on [Akkouchi 2008, Jasiulewicz and Kordecki 2003] random variable T_l has thickness specified by the equation:

$$f_{T_l}(\tau) = \sum_{i=1}^{n} \lambda_{l,i}^{\ k} \cdot e^{-\tau\cdot\lambda_{l,i}} \sum_{j=1}^{k} \frac{(-1)^{k-j}}{(j-1)!}\tau^{j-1} \sum_{\substack{m_1+\ldots+m_n=k-j \\ m_i=0}} \prod_{\substack{h=1 \\ h\neq i}}^{n} \binom{k+m_h-1}{m_h} \frac{\lambda_{l,h}^{\ k_l}}{(\lambda_{l,h}-\lambda_{l,i})^{k+m_h}}$$

(12)

where: $\lambda_{l,i}$ – parameter of exponential composition, k – parameter of the shape of the Erlang composition, m_1 , $m_2, \ldots, m_n \in$ N – parameters of combination when m_i=0 oraz $\sum_{g=1}^{n} m_g = k - j$ for all $i = 1, 2, \ldots, n$ and $j = 1, 2, .., k$. Medium of random variable T_l is $\tau > 0$.

Expected value of random variable T_l is:

$$ET_l = \sum_{i=1}^{n} \frac{k}{\lambda_{l,i}} \tag{13}$$

Expected value of variable T which sets whole k time of final products is formed:

$$ET = \sum_{l=1}^{m}\sum_{i=1}^{n} \frac{k}{\lambda_{l,i}} \tag{14}$$

To be able to assign dynamics of changes of SP for production party $2 \le k \le 30$ in the next step there will be set variant of random variable T. Using independency of random variables $t_{l,i}$ for $i = 1, 2, \ldots, n$ we receive:

$$D^2T_l = D^2t_{l,1} + D^2t_{l,2} + \ldots + D^2t_{l,n} = \sum_{i=1}^{n} \frac{k}{\lambda_{l,i}^{\ 2}} \tag{15}$$

So, variant of random variable T which is a sum of independent random variables T_l can be presented with the equation:

$$D^2T = \sum_{l=1}^{m}\sum_{i=1}^{n} \frac{k}{\lambda_{l,i}^{\ 2}} \tag{16}$$

In the article there is a presentation of the task in which shaping of dynamics in changes of convergent production system depends on the size of production party. In next part there is a presentation of a model which describes variation for system of $k > 30$ stocks.

Shaping of a realization time of order for $k > 30$

In area of statistics, each sample which is bigger than 30 can be considered big. Basing on that, and using Central Limit Theorem (CLT) we can provide approximate number (with the usage of normal composition) [Devore 2012; Lange 2010].

In our consideration, we can assume that k depends on number of orders and it is deterministic value. Moreover, we assume that k is the same for all of i – processes. Then, random variable T_l which sets a sum of realization of n processes in l value stream for k stocks necessary to finish the tasks (elements) can be presented by the equation:

$$T_l = t_{l,1} + t_{l,2} + \ldots + t_{l,n} \tag{17}$$

where:

$$t_{l,i} = \tau_{l,1} + \tau_{l,2} + \ldots + \tau_{l,n} \tag{18}$$

where: $t_{l,i} \sim Erl\,(k, \lambda_{l,i})$, same interpretation as for $2 \le k \le 30$.

Basing on the fact that random variables $t_{l,i}$ are the sum of independent random variables $\tau_{l,i,j}$ with the same exponential composition – $\tau_{l,i,j} \sim \mathrm{Exp}\,(\lambda_{l,i})$; we can use Central Limit Theorem (CTG) to provide approximate random variable $t_{l,i}$ (using normal composition). Then:

$$t_{l,i} \overset{CTG}{\sim} N\left(k\cdot\frac{1}{\lambda_{l,i}}, \frac{1}{\lambda_{l,i}}\cdot\sqrt{k}\right) \quad \forall \quad i = 1, 2, \ldots, n \tag{19}$$

Using above dependency and the fact that:

$$T_l = t_{l,1} + t_{l,2} + \ldots + t_{l,n} \tag{20}$$

we receive:

$$T_l \sim N\left(\sum_{i=1}^{n} \frac{k}{\lambda_{l,i}} , \sqrt{\sum_{i=1}^{n} \frac{k}{\lambda_{l,i}^{\;2}}} \right) \tag{21}$$

finally, random variable T sets whole time for realization of k tasks:

$$T \sim N\left(\sum_{l=1}^{m} \sum_{i=1}^{n} \frac{k}{\lambda_{l,i}} , \sqrt{\sum_{l=1}^{m} \sum_{i=1}^{n} \frac{k}{\lambda_{l,i}^{\;2}}} \right) \tag{22}$$

Above indicates:

$$ET = \sum_{l=1}^{m} \sum_{i=1}^{n} \frac{k}{\lambda_{l,i}} \tag{23}$$

and:

$$D^2 T = \sum_{l=1}^{m} \sum_{i=1}^{n} \frac{k}{\lambda_{l,i}^{\;2}} \tag{24}$$

where formula (23) sets expected value for random variable T and basing on formula (24) we can set variant of this random variable.

To set dynamics of convergent changes of production system which depends on size of production party, in the next part there will be set a probability that random variable will take value distant from expected value ET. There might be a difference of maximum 3 in standard deviation and results will be compared with three sigma rule ($\pm 3\sigma$) for normal composition.

Analysis of three sigma – 3σ

Three sigma rule for normal composition is strictly connected to standard deviation σ. This rule informs us that in interval $(ET - 3 \cdot DT ; ET + 3 \cdot DT)$ there is around 99,7% of all observations for random variable of normal composition, in which standard deviation is marked by DT. When random variable has different composition than normal – interval is changed. Basing on Chebyshev inequality we can assume that for each composition [Durrett 2010]:

- minimum 75% of observations is allocated around $\pm 2\ DT$;
- minimum 88,9% of observations is allocated around $\pm 3\ DT$;
- minimum 93,7% of observations is allocated around $\pm 4\ DT$.

In next considerations, we will assume that random variable T, (described in chapter 2.1) takes value in the interval $(ET - 3 \cdot DT ; ET + 3 \cdot DT)$.

Basing on fact that random variable T_l has thickness specified by the formula (12) and a fact that each of T_l variables is a sum of all variables of Erlang composition, which pairs have different parameters $\lambda_{l,i}$ for all processes and all streams we will receive thickness of variable T. We have to conduct change of indexes of all $\lambda_{l,i}$ parameters. After segregating all parameters in one line, we receive: $\lambda_{1,1} ; \lambda_{1,2} ; \ldots ; \lambda_{1,n} ; \ldots ; \lambda_{m,1} ; \lambda_{m,2} ; \ldots ; \lambda_{m,n}$. Now we have to mark all parameters from first stream by: $\lambda_1 ; \lambda_2 ; \ldots ; \lambda_n$; second by: $\lambda_{n+1} ; \lambda_{n+2} ; \ldots ; \lambda_{2n}$; until m – stream $\lambda_{(m-1)n+1} ; \ldots ; \lambda_{mn}$. As a consequence, we receive $m \cdot n$ different paramters λ. That is why, thickess of random variable T will be set by the formula:

$$f_T(\tau) = \sum_{i=1}^{m \cdot n} \lambda_i^{\,k} \cdot e^{-\tau \cdot \lambda_i} \sum_{j=1}^{k} \frac{(-1)^{k-j}}{(j-1)!} \tau^{j-1} \cdot \sum_{\substack{s_1 + \ldots + s_{(m \cdot n)} = k-j \\ s_i = 0}} \prod_{\substack{h=1 \\ h \neq i}}^{m \cdot n} \binom{k + s_h - 1}{s_h} \frac{\lambda_h^{\,k}}{(\lambda_h - \lambda_i)^{k+s_h}} \tag{25}$$

Using above formula, we can count probability accordingly:

$$P(ET - 3 \cdot DT < T < ET + 3 \cdot DT) = \int_{ET - 3 \cdot DT}^{ET + 3 \cdot DT} f_T(\tau)\, d\tau \tag{26}$$

Using $f_T(\tau)$ with *ET* and *DT* we receive:

$$P\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}<T<\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}+3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)= \tag{27}$$
$$=\int_{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}}^{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}+3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}}\left\{\sum_{i=1}^{m\cdot n}\lambda_i^k\cdot e^{-\tau\cdot\lambda_i}\sum_{j=1}^{k}\frac{(-1)^{k-j}}{(j-1)!}\cdot\tau^{j-1}\cdot\sum_{\substack{s_1+\ldots+s_{(m\cdot n)}=k-j\\ s_i=0}}\prod_{\substack{h=1\\ h\neq i}}^{m\cdot n}\binom{k+s_h-1}{s_h}\frac{\lambda_h^k}{(\lambda_h-\lambda_i)^{k+s_h}}\right\}d\tau$$

In other changes we receive difference of incomplete Gamma functions [Di Salvo 2006]:

$$P\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}<T<\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}+3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)=$$
$$=\sum_{i=1}^{m\cdot n}\sum_{j=1}^{k}\lambda_i^{k-j}\cdot\frac{(-1)^{k-j}}{(j-1)!}\cdot\sum_{\substack{s_1+\ldots+s_{(m\cdot n)}=k-j\\ s_i=0}}\prod_{\substack{h=1\\ h\neq i}}^{m\cdot n}\binom{k+s_h-1}{s_h}\frac{\lambda_h^k}{(\lambda_h-\lambda_i)^{k+s_h}} \tag{28}$$
$$\cdot\left[\Gamma\left(j,\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)\lambda_i\right)-\Gamma\left(j,\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}+3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)\lambda\right)\right]$$

Finally, after changes we receive:

$$P\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}<T<\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}+3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)=$$
$$=\sum_{i=1}^{m\cdot n}\sum_{j=1}^{k}\frac{(-\lambda_i)^{k-j}}{(j-1)!}\cdot\sum_{\substack{s_1+\ldots+s_{(m\cdot n)}=k-j\\ s_i=0}}\prod_{\substack{h=1\\ h\neq i}}^{m\cdot n}\binom{k+s_h-1}{s_h}\frac{\lambda_h^k}{(\lambda_h-\lambda_i)^{k+s_h}}\cdot$$
$$\cdot(j-1)!\cdot e^{-\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}\cdot\lambda_i}\cdot\left[\sum_{r=0}^{j-1}\left(e^{3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\cdot\lambda_i}\cdot\frac{\left(\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)\cdot\lambda_i\right)^r}{r!}-e^{-3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\cdot\lambda_i}\cdot\frac{\left(\left(\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i}+3\cdot\sqrt{\sum_{i=1}^{m\cdot n}\frac{k}{\lambda_i^2}}\right)\cdot\lambda_i\right)^r}{r!}\right)\right] \tag{29}$$

RESULTS

In the article there is a presentation of models which can be used to set dynamics of changes in complex, convergent production structure. Wrought models take into consideration randomization of variables in times of cycles in realizing processes which depend on a type of final product. Models take into consideration size of production party and implemented technology. As a result of wrought models – we receive formulas which can be implemented in IT tolls (e.g. R Studio or Witness System Simulation Modeling) environment. Those are setting function of thickness of probability of times of single streams (Tl) and function of thickness of probability of times of all available streams, determined by implemented technology (T).

In the article, there is also a formula which has been led out to set probability of the fact that random variables will be allocated in area of three in standard deviation (in comparison to designated expected value). For considered production structure, there were some pilot calculations conducted. In those, the minimum result for short series of production (up to 1 stock) was around 98,2%. Other examples of values – 98,6 % for 2 stocks; 99 % for 5 stocks and 99,3% for 10 stocks. So initial calculations for considered example confirm that size of production party has impact on stability of production structure. For calculations in which only times of processes have changed – results were the same. Different values of probability have been achieved in a case of changes in technology. Basing on received results, we can state that time of cycle do not influence stability of production structure. Authors are quite moderate in terms of this hypothesis (without conducting proper, statistically big series of calculations with adjusting changes in technology). Changes in technological creation of product are directly connected to the changes in the structure of certain sub-systems [Zimon and Malindžák 2017]. Further works will implement presented models in available IT tool – to be able to validate considered production structure. Moreover, problem of lack in stability of production of MTO type will be a base for further research on multi-criterial function of optimization for complex, convergent production systems.

CONCLUSIONS

The main purpose of this article was to present changes in dynamics of convergent production system. In presented consideration, dynamics of changes is considered as difference of expected value of random variable which sets realization time of all necessary production tasks for different amount of production party. Presented models of shaping the dynamics can help with a balance of production streams and early warnings before appearance of critical situation. In practice, results different than $\pm 3\sigma$ are extremely rare. However it does not mean that their appearance is impossible. Situation in which this kind of difference occurs, means that the circumstances which appeared were extremely rare as well and also they can be a sign of anomaly. Further development of presented model will be set to indicate optimal parameters of function of the thickness probability in random variable which sets realization time for tasks to present deterministic results of D2T and DT with relatively lowest values.

Ideally balanced stream of values is one in which particular single expected values in different production sockets are similar (or even equal) with expected value of whole production system. That is why it is so important to keep stability of ET for singular sub-system in comparison to all production structure. This task is quite difficult because in convergent structures on particular transmission steps there are different values of times for tasks. To be able to achieve that, it is important to use dynamic steering and scheduling of production processes [Grzybowska et al. 2012, Lenart et al. 2012].

Presented models after implementing them in IT environment will be used to validate real objects. Lacks in stability of production system can result in generating bigger operational costs, increase of the works in process (WIP) which can generate overproduction and appearance of 3M waste ("muri", "mura" and "muda").

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KSZTAŁTOWANIE DYNAMIKI ZMIAN KONWERGENTNEGO SYSTEMU WYTWÓRCZEGO W ZALEŻNOŚCI OD WIELKOŚCI PARTII PRODUKCYJNEJ

STRESZCZENIE. Wstęp: Koncepcja Lean Six Sigma skupia się na kliencie i jego potrzebach. Istnienie potrzeb klientów generuje podaż przedsiębiorstw. Przedsiębiorstwa produkujące zgodnie z MTO (make-to-order) muszą charakteryzować się dużą elastycznością i dostosowaniem do dynamicznych zmian otoczenia (w tym zmian zapotrzebowania klienta). Daleko idąca customiazacja ma negatywny wpływ na strukturę systemu produkcyjnego. Występująca dynamika zmian, brak przewidywalności stanu systemu w chwili t+1 skutkuje zwiększeniem kosztów operacyjnych. W szczególności dotyczy to tych przedsiębiorstw, które produkują zróżnicowany asortyment, w krótkich seriach produkcyjnych. Celem naukowym artykułu jest opracowanie modelu matematycznego określającego poziom zmienności struktury systemu produkcyjnego w zależności od wielkości partii produkcyjnej dla układu wytwórczego zgodnego z MTO. Ponadto w artykule zaprezentowane zostały pilotażowe obliczenia określające wartość prawdopodobieństwa, jak występujące zmienne losowe mieszczą się w obszarze trzech odchyleń standardowych (±3δ) od wyznaczonej wartości oczekiwanej (ET) dla całej struktury produkcyjnej. Zaprezentowane modele są rezultatem analiz i wniosków wielomiesięcznych prac związanych z wdrażaniem wybranych narzędzi lean toolbox w jednym z polskich przedsiębiorstw sektora MSP. Struktura produkcyjna rozpatrywanego obiektu rzeczywistego jest złożona i ma charakter konwergentny, zgodny MTO przy czym wyroby finalne wytwarzane są w krótkich seriach produkcyjnych przy względnie bardzo wysokiej customizacji produktów.

Materiały i wyniki: Opracowane modele uwzględniają rozważania ujęcia systemowego zgodnie z ogólną teorią systemów według Klira oraz Meserovicza [Mesarovic 1964] jak również teorię obsługi masowej będących jednym z działów teorii prawdopodobieństwa. W artykule przedstawione są dwa modele, które stanowią podstawowy argument w definiowaniu problemów z zakresu inżynierii logistyki [Michlowicz et al. 2015] i produkcji w badaniach naukowych. Ważnym atrybutem przedstawionych modeli jest fakt, iż uwzględniają one zależności występujących zmiennych losowych w strukturze wykonywania następujących po sobie poszczególnych procesów oraz uwzględniają zależności wielkości partii produkcyjnej dla obiektu rzeczywistego.

Przedstawione modele nie są jedynie opracowaniem teoretycznym ale uwzględniają zależności rzeczywiste i empiryczne. Rozważaniom został poddany rzeczywisty obiekt wytwórczy, specjalizujący się w produkcji urządzeń chłodniczych przeznaczonych do przechowywania preparatów krwiopochodnych oraz osocza i krioprecypitatu. Urządzenia te posiadają bardzo restrykcyjne wymogi jakościowe, zgodne z ISO 13485 (Systemy Zarządzania Jakością dla Wyrobów Medycznych) oraz znakiem CE0434 (dla urządzeń spełniających warunki Dyrektywy 93/42/EEC).

W artykule zostały przedstawione dwa modele wyznaczające funkcję czasów produkcji (VA – Value Added) dla przypadku gdy partia produkcyjna wynosi sztuk tego samego wyrobu oraz gdy partia produkcyjna wynosi sztuk. W opracowanym modelu zostały uwzględnione następujące parametry systemu produkcyjnego: zmienne parametry czasów trwania procesów zależne od rodzaju wytwarzanego półproduktu, zależność wartości czasów trwania od wielkości partii produkcyjnej, zależność czasów operacji od zaimplementowanej technologii.

Wnioski: W opracowywaniu rozwiązań, które implementowane są w obiektach rzeczywistych ważne jest dostosowanie narzędzi do indywidualnych cech usprawnianego systemu. To co jest korzystne w jednej organizacji nie zawsze jest efektywne w innym przedsiębiorstwie. Opracowany model (uwzględniający zależności realizujących jedynie zlecenia w tzw.: produkcji jednostkowej) jest pierwszym z etapów budowy układu służącego do walidacji rzeczywistego obiektu dla chwili t+1. W kolejnym etapie przeprowadzone rozważania zostaną zaimplementowane w środowisku narzędzia informatycznego R Studio w celu przeprowadzenia analiz statystycznych na podstawie danych historycznych.

Słowa kluczowe: Konwergentny system produkcyjny, model dynamiki zmian

GESTALTUNG DER DYNAMIK DER ÄNDERUNGEN DES KONVERGENTEN HERSTELLUNGSPROZESSES IN ABHÄNGIGKEIT VON DER GRÖSSE DER HERSTELLUNGSPARTIE

ZUSAMMENFASSUNG. Einleitung: Das Konzept von Lean Six Sigma konzentriert sich auf den Kunden und seine Bedürfnisse. Das Bestehen der Kundenbedürfnisse generiert das Angebot der einzelnen Unternehmen. Die Unternehmen, die nach dem MTO-Prinzip (make-to-order) Produkte herstellen, müssen sich durch eine große Flexibilität und Anpassung an dynamische Umfeldveränderungen (dabei auch die Veränderungen des Kundenbedarfs) charakterisieren. Die weitgehende Customerisierung hat einen negativen Einfluss auf die Struktur des Herstellungssystems. Die auftretende Änderungsdynamik, Unmöglichkeit, den Systemzustand in der Zeit t+1 vorzusehen, hat die Steigerung von operativen Kosten zur Folge. Dies betrifft vor allem die Unternehmen, die ein unterschiedliches Sortiment in kurzen Produktionsserien erzeugen. Das wissenschaftliche Ziel dieses Artikels ist die Bearbeitung eines mathematischen Modells, mit dem sich das Änderungsniveau der Struktur eines Herstellungssystems bestimmen lässt, je nach

Serienmenge für ein mit dem MTO übereinstimmendes Fertigungssystem. Außerdem werden im Artikel versuchsweise die den Wahrscheinlichkeitswert bestimmenden Berechnungen zur Feststellung von auftretenden Zufallsvariablen im Bereich von drei Standardabweichungen (±3δ) vom festgesetzten Erwartungswert (ET) für die ganze Herstellungsstruktur dargestellt. Die präsentierten Modelle sind Ergebnisse von mehrere Monate lang andauernden Analysen und Schlussfolgerungen, die mit dem Einsatz der ausgewählten Lean Toolbox- Mittel im Sektor der polnischen MSP-Unternehmen (Sektor von kleinen und mittleren Unternehmen) verbundenen sind. Die Herstellungsstruktur des analysierten wahren Objektes ist vielfältig und weist auf einen konvergierten, mit dem MTO in Übereinstimmung stehenden Charakter hin, wobei Endprodukte in kurzen Serien und bei einer streng individuell angepassten Gestaltung erzeugt werden.

Methoden: Die bearbeiteten Modelle berücksichtigen die Erwägungen der Systemauffassung gemäß der allgemeinen Systemtheorie nach Klir und Meserovicz [Mesarovic 1964], sowie die Theorie der Massenbedienung - beide sind ein Teil der Wahrscheinlichkeitstheorie. In dem Artikel werden zwei Modelle dargestellt, die das grundlegende Argument bei dem Definieren der Probleme im Bereich der Logistik [Michlowicz et al. 2015] und der Herstellungsingenieurkunde in den wissenschaftlichen Forschungen bilden. Ein wichtiges Attribut der dargestellten Modelle ist die Tatsache, dass sie die Abhängigkeiten der auftretenden Zufallsvariablen in der Realisierungsstruktur der aufeinanderfolgenden einzelnen Prozesse und die Abhängigkeiten der Größe der Herstellungspartie für ein wirkliches Objekt berücksichtigen.

Die dargestellten Modelle bilden nicht nur eine theoretische Bearbeitung, sondern berücksichtigen auch die wirklichen und empirischen Abhängigkeiten. Den Erwägungen wurde ein wirkliches Produktionsobjekt, das sich auf die Herstellung von Kühleinrichtungen zur Aufbewahrung von Blutpräparaten sowie des Plasmas und Kriopräzipitats spezialisiert, unterzogen. An solche Einrichtungen werden sehr restriktive Qualitätsanforderungen gemäß ISO 13485 (Qualitätsmanagementsysteme für medizinische Erzeugnisse) und dem Symbol CE0434 (für Einrichtungen, die den Bedingungen der Richtlinie 93/42/EWG entsprechen) gestellt.

In dem Artikel wurden zwei Modelle dargestellt, die die Funktion der Herstellungsdauer (VA - Value Added) für den Fall projizieren, wenn die Herstellungspartie Stück desselben Produktes beträgt und wenn die Herstellungspartie Stück beträgt. In dem bearbeiteten Modell wurden folgende Parameter des Produktionssystems berücksichtigt: veränderliche Parameter der Prozessdauer, die von der Art des hergestellten Halbproduktes abhängig sind, Abhängigkeit der Zeitdauer von der Größe der Herstellungspartie, Abhängigkeit der Operationsdauer von der implementierten Technologie.

Fazit: Bei der Bearbeitung der Lösungen, die in die wirklichen Objekte implementiert werden, ist es wichtig, die Werkzeuge den individuellen Merkmalen des zu verbessernden Systems anzupassen. Das, was innerhalb einer Organisation von Vorteil ist, ist in einem anderen Unternehmen nicht immer effektiv. Das bearbeitete Modell, das nur die Abhängigkeiten bei der Realisierung von Aufträgen in der sog. Einheitsproduktion berücksichtigt, ist die erste Etappe zum Aufbau des Systems zur Validation des wirklichen Objektes in der Zeit t+1. In der nächsten Etappe werden die durchgeführten Erwägungen in das Umfeld des informatischen Werkzeugs R-Studio zur Realisierung der statistischen Analysen an Hand von historischen Daten implementiert.

Codewörter: Konvergentes Herstellungssystem, Modell der Änderungsdynamik

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Bożena Zwolińska
AGH University of Science and Technology
al. Mickiewicz 30, 30-059
Kraków, **Poland**
e-mail: bzwol@agh.edu.pl

Łukasz Kubica
Jagiellonian University in Kraków
ul. Gołębia 24 30, 31-007
Kraków, **Poland**
e-mail: lukasz.kubica@im.uj.edu.pl