THEORETICAL INVESTIGATIONS OF THREE-NUCLEON SYSTEMS*

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We present selected results of theoretical investigations of three-nucleon systems which have been pursued in Kraków for more than thirty years. The Kraków–Bochum group has gathered a lot of experience related to investigations of elastic nucleon–deuteron scattering and nucleon-induced deuteron breakup processes. These investigations are based on rigorous solutions of the 3N Faddeev equations in momentum space and aim to understand the properties of two- and three-nucleon forces. Since the late 1980s, very many different models of nuclear potentials, including several generations of the forces derived within chiral effective field theory, have been put to stringent tests. Beside pure three-nucleon reactions, our Faddeev framework has been used to describe many electroweak processes, where the initial-state or final-state interactions among three-nucleons are very important. Our theoretical results need to be confronted with precision experimental data, so collaboration with many experimental groups all over the world is crucial for our research.

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1. Introduction

Understanding the structure of the nuclear Hamiltonian is the central challenge in nuclear physics. The necessity for the three-nucleon force (3NF) was established when three-nucleon (3N) bound states were calculated exactly [1–3] using the early "realistic" nucleon–nucleon (NN) potentials [4–8]

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and later employing semi-phenomenological NN potentials which described the NN data set with high precision (χ^2/datum ≈ 1) [9–11]. These findings were subsequently confirmed by calculations of the four-nucleon (4N) bound state [12, 13]. The observed underbinding of the 3N and 4N bound states was explained by augmenting the nuclear Hamiltonian with a 3NF, such as the Tucson–Melbourne (TM) [14] or the Urbana IX [15] model.

The bound states did not provide sufficient information about the properties of the 3N Hamiltonian and the 3N scattering states became mandatory to shed more light on this problem. The proper mathematical foundations for 3N scattering were formulated already in the 1960s by Faddeev [16, 17] and later given in many different forms, for example as the Alt–Grassberger–Sandhas (AGS) equations [18]. However, as stated in Ref. [18], any direct solution of the Faddeev equations for a long time revealed "nearly insurmountable calculational difficulties". A breakthrough took place in the late 1980s when Henryk Witała received a Humboldt Research Fellowship for postdoctoral researchers and professor Walter Glöckle from the Ruhr University in Bochum became his academic host in Germany. The two scientists worked very hard, pushing available computer resources to their limits, and managed to develop for the first time a set of numerical algorithms and programs that were subsequently used by them to obtain *exact* numerical solutions of the 3N continuum Faddeev equations with realistic NN forces. This great achievement gave a solid foundation for a theoretical interpretation of experimental data and for studies of various ingredients in the nuclear Hamiltonian, without introducing any uncontrolled approximations. In particular, in the early 1990s, numerical solutions of the 3N Faddeev equations for nucleon–deuteron scattering with inclusion of realistic 3N forces became available.

This had great impact on the field of few-nucleon physics and triggered vivid activities of experimental groups in Kraków, Katowice (Poland), PSI (Switzerland), Bochum, Bonn, Cologne, Erlangen (Germany), University of Tokyo, RIKEN, RCNP, Kyushu (Japan), KVI (The Netherlands) and TUNL, IUCF Bloomington (USA). Calculations performed in the Cracow–Bochum group enabled experimentalists to prepare measurements sensitive to specific features of the nuclear Hamiltonian and study the role of particular NN force components, charge independence breaking, the structure of the 3N force. All these investigations and the progress achieved by other theoretical groups proved for the first time that nuclear physics could be understood as a theory of nucleons interacting with two- and three-body forces arising from meson exchanges. Many important results obtained before the mid-1990s for the 3N system were summarized in a review paper [19], which is a crucial reference for anyone interested in 3N calculations.

2. Nucleon–deuteron scattering with semi-phenomenological nuclear potentials

These calculations of the elastic nucleon–deuteron scattering reaction and the nucleon-induced deuteron breakup process carried out with the semi-phenomenological forces [9–11] showed (see, for example, Refs. [19–21]) that, in general, predictions for 3N scattering observables agree well with data at the incoming nucleon energies below approximately 30 MeV. At higher energies, however, clear discrepancies between the theoretical predictions based on NN forces only and data were observed. For the minimum of the elastic scattering cross section agreement with the data was recovered for energies below approximately 140 MeV, when the 3NF models [14, 15], whose parameters were adjusted to reproduce the experimental triton binding energy, were additionally employed in the 3N calculations [20–23]. In Fig. 1, we show predictions for two different nucleon laboratory energies, 65 and 135 MeV, only for one combination of the NN and 3N forces but the picture remains true also for the other models of nuclear interactions.

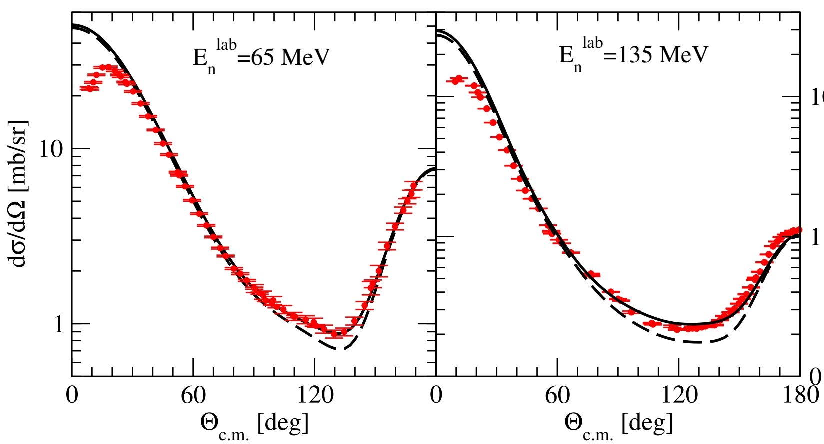

Fig. 1. The elastic differential nucleon–deuteron cross section as a function of the center-of-mass scattering angle Θ_{cm} corresponding to the laboratory nucleon energy $E_n^{\mathrm{lab}} = 65$ MeV (left panel) and 135 MeV (right panel). While predictions obtained with the CD Bonn NN potential alone are shown with dashed curves, results of full calculations employing additionally the TM 3NF are represented by solid lines. The data come from Ref. [25] (left panel) and from Ref. [26] (right panel).

For many spin observables in elastic nucleon–deuteron scattering (for example, the nucleon analyzing power and the deuteron tensor analyzing powers [21, 24]), large 3NF effects were predicted, but the available combinations of 2N and 3N forces could not describe the data. One of possible reasons for this disagreement between the theoretical results and the data could be the lack of relativity in the formalism. However, the results obtained within the framework of relativistic Faddeev equation [27, 28] showed only small effects in the cross section. In addition, elastic scattering polarization observables were only slightly changed by relativity at the considered energies [27, 28].

All these studies led to the conclusion that the discrepancies observed at higher energies, which could not be removed when the Tucson–Melbourne or Urbana IX 3NF models were included in the calculations, required 3NF models with a more sophisticated spin structure, containing also short-range components, and consistence between the 2N and 3N potentials. This could be achieved only within the chiral effective field theory.

3. Nucleon–deuteron scattering with chiral nuclear potentials

In [29], for the first time, low energy 3N scattering was studied with chiral next-to-next-to-leading order (N2LO) 2N and 3N forces. In Refs. [30, 31], 2N potentials were developed at next-to-next-to-next-to-leading order (N3LO) of the chiral expansion. They could be used in a wider energy range and described experimental phase-shifts [32, 33] as well as the semi-phenomenological 2N potentials. The N3LO contributions to the 3NF derived in Refs. [34, 35] do not contain any additional unknown parameters so the full N3LO 3NF possesses only two low-energy constants. In order to fix these free parameters, at least one 3N scattering observable is required, in addition to the experimental binding energy of ^{3}H, for example the nucleon–deuteron doublet scattering length. Recently, the very precise experimental data for the proton–deuteron differential cross section at the proton laboratory energy of 70 MeV from Ref. [26] is also chosen. The first generation chiral 2N and 3N forces were tested in the calculations of the elastic nucleon–deuteron scattering observables [36]. We found that non-local regularization applied directly in momentum space led to strong finite-cutoff artefacts in the results for higher-energies. This precluded employing such forces in 3N continuum calculations.

The Kraków–Bochum group's calculations played also an important role in testing more recent versions of 2N chiral potentials prepared up to next-to-next-to-next-to-next-to-leading order (N4LO). While the first of them [37, 38] employed a local coordinate-space regularization of the one- and two-pion exchange contributions, the newest one from Ref. [39] uses a momentum-space version of the local regulator. The change of the regularization strategy led to a substantial reduction of the finite-cutoff artefacts for high-energy observables, especially for the differential cross section.

The new NN potentials [37–39] are developed and tested within the Low Energy Nuclear Physics International Collaboration (LENPIC), which in particular "aims to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding exchange currents" [40]. This initiative gathered physicists from several institutions: Ruhr-University Bochum, Germany, University of Bonn, Germany, Technical University of Darmstadt, Germany, Jagiellonian University, Kraków, Poland, Iowa-State University, USA, Jülich Research Centre, Germany, Kyushu Institute of Technology, Japan, Ohio State University, USA, Orsay Institute of Nuclear Physics, France, and TRIUMF, Canada. The Co-Spokespersons of this collaboration are Prof. Evgeny Epelbaum from Ruhr-University Bochum and Prof. James Vary from Iowa State University. Results obtained by LENPIC members in few-nucleon and many-nucleon systems [41–43] with the NN potentials alone demonstrate clearly the need of 3NF contributions. The implementation of a consistent local regulator turned out to be very complicated for the 3N potentials beyond N2LO, so in Refs. [44, 45], the consistent study of nucleon–deuteron scattering as well as ground and low-lying excited states of nuclei with $A \leq 16$ was performed only up to N2LO.

All the studies mentioned in this section prove the high quality of the newest chiral NN potentials [39]. The studies including the consistently regularized N2LO 3N force are also very encouraging [45]. However, any further progress requires consistently regularized 3N forces at least at N3LO. This work is in progress by the LENPIC Collaboration.

4. Electroweak processes

Despite many open questions in the pure 3N system, it became clear that the methods and computer codes developed by our Kraków–Bochum group could be applied to various electroweak processes, where 3N continuum appears in the initial or in the final state. The crucial nuclear matrix elements of the corresponding current operators are constructed from solutions of the Faddeev-like equation, which has the same kernel as the original one appearing for the nucleon–deuteron scattering reaction. We started with electron- and photon-induced break-up of ^{3}He (^{3}H) and the closely related nucleon–deuteron radiative capture. The set of codes for exclusive, semi-exclusive and fully inclusive reactions was used to analyze experimental data from NIKHEF (The Netherlands), MIT Bates, Jefferson Lab, TUNL (USA), Mainz (Germany) and Lund (Sweden). Many results for these electromagnetic reactions obtained with the semi-phenomenological nuclear forces and current operators can be found in our review paper [46].

Later, we extended our investigations to various electroweak processes: non-mesonic and mesonic decays of the hypertriton [47, 48], muon capture on ^{3}He and ^{3}H [49, 50], (anti)neutrino scattering off ^{3}He and ^{3}H [51, 52], and pion radiative capture in ^{3}He and ^{3}H [53], using the framework of Ref. [46]. Although our calculations leave room for improvement, we provided important predictions for all the above-mentioned reactions. As an example, we show in Fig. 2 our results from Ref. [52] for the total breakup cross section in three inclusive (anti)neutrino reactions with ^{3}H.

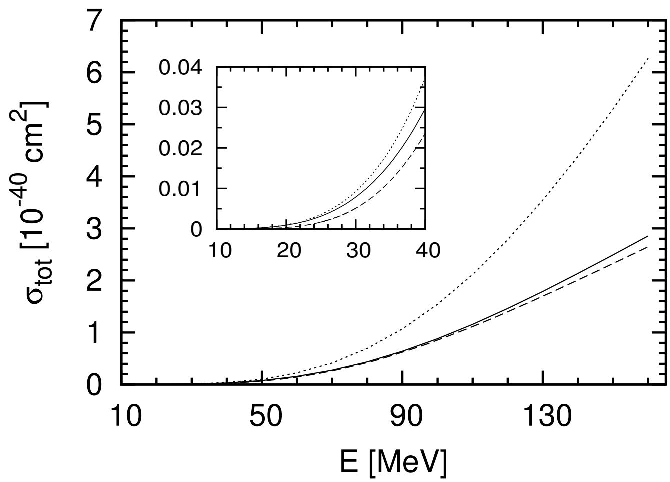

Fig. 2. The total breakup cross section for inclusive charged-current electron antineutrino disintegration of ^{3}H (dashed line), neutral-current electron antineutrino disintegration of ^{3}H (solid line), and neutral-current electron neutrino disintegration of ^{3}H (dotted line) as a function of the laboratory (anti)neutrino energy E. The figure was originally published in [52].

All electroweak processes should be ultimately studied employing single-nucleon and many-nucleon electromagnetic and axial current operators which are built consistently with the nuclear forces. Although electromagnetic and axial currents have been derived in chiral effective field theory completely up to N3LO [54–57], they are not yet regularized and thus cannot be used in practical calculations together with the nuclear forces employing the local regularization.

That is why selected electromagnetic processes with real photons were studied in Ref. [58] using the Siegert theorem. In this way, we could include implicitly certain types of many-nucleon contributions in the nuclear current operator. The results obtained with the chiral potentials employing the local coordinate-space regularization showed a very weak dependence of predictions on the cutoff parameter and provided a good data description.

We plan to apply chiral currents to low-energy photodisintegration processes and muon capture from the lowest atomic orbits, where the energy and momentum transfers are clearly limited. We hope that calculations based on chiral forces and currents will be important for the results of electron scattering experiments planned at the MESA accelerator in Mainz, Germany.

5. "Three-dimensional" calculations

In parallel to the work presented in the previous sections, the Kraków group has been developing alternatives to traditional calculation schemes that are based on partial waves. These new, so-called, "three-dimensional" (3D) calculations work directly with the 3D degrees of freedom of the nucleons: the momentum eigenstates. Using this approach means that the 3D calculations, at least in principle, incorporate all possible partial waves at once. This will allow us to perform very precise calculations, especially related to nuclear systems at higher energies, but this ability is limited by the available computational resources.

In order to decrease the computational workload of the 3D calculations, we employ operator forms of states (see *e.g.* [59, 60]) and operators (see *e.g.* [60, 61]). Using these forms, the fundamental (Schrödinger, Lippmann–Schwinger, Faddeev) equations are reduced to relations governing scalar functions of relevant momenta (for the 2N case — the relative momentum of the nucleons, and for the 3N case — the Jacobi momenta). Once these scalar functions are calculated, they can be used to recreate the object of interest (wave function, Faddeev component or transition operator). One recent development of the Kraków group is a new operator form of the 3N scattering amplitude [63] that opened up the possibility to perform 3D calculations describing both, the elastic and breakup, channels of nucleon deuteron scattering.

So far, the 3D approach has been successfully applied to describe a number of few-nucleon systems. 3D calculations of the deuteron [60] can utilize any 2N potential that satisfies the usual symmetries of parity, time reversal and particle exchange. Calculations of the ^{3}H and ^{3}He [59, 62] Faddeev component and wave function can in addition to the 2N potential use a very general form of the 3N force. Moreover, 2N scattering can already be very well described using the 3D calculations [60]. Recent work, motivated by encouraging first order results [64], is focused on calculating the 3N scattering amplitude from the full solution of the Faddeev equation. We believe that this is possible with the new operator form of the amplitude [63] and that this new tool will enable us to test nuclear forces in more challenging kinematical regimes. Additionally, the possibility to skip the partial decomposition procedure will make these calculations more flexible.

6. Summary

We described briefly results of our theoretical investigations of three-nucleon systems carried out in Kraków since the late 1980s. These investigations are based on rigorous solutions of the 3N continuum Faddeev equations in momentum space for energies below the pion production threshold. The calculations started with the description of cross sections and many polarization observables in elastic nucleon–deuteron scattering as well as nucleon-induced deuteron breakup processes. Later, a modified Faddeev framework was used to analyze various reactions of electroweak probes with 3N systems. Our aim has been always the same: to understand the structure of nuclear forces and (later) the properties of the electroweak current operators. The investigations of the 3N force effects have been especially important and led to very intense collaboration with many experimental groups. Recently, we have placed particular emphasis on studies of the forces derived within chiral effective field theory within the LENPIC initiative. Work on applications of the N3LO 3N forces and consistent current operators is in progress.

This work is a part of the LENPIC project and we thank other LENPIC members for sharing with us their expertise in the field of chiral forces and current operators discussed in this contribution. This work was supported by the National Science Centre, Poland (NCN) under grants Nos. 2016/22/M/ST2/00173 and 2016/21/D/ST2/01120. The numerical calculations were partially performed on the supercomputer cluster of the JSC, Jülich, Germany.

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