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Saturation and geometrical scaling – from small x deep inelastic ep scattering to high energy proton-proton and heavy ion collisions

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Gluon distributions of colliding hadrons saturate as a result of the non-linear evolution equations of QCD. As a consequence there exists the so called saturation momentum, which is related to the gluon density per unit rapidity per transverse area. When saturation momentum is the only scale for physical processes, different observables exhibit geometrical scaling (GS). We show a number of examples of GS and its violation in different reactions.

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1. Deep inelastic scattering

In this talk we summarize, following Ref. [1], our recent studies of geometrical scaling in high energy collisions. Deep inelastic scattering (DIS) is well described in terms of the dipole model (see *e.g.* [2] and references therein):

$$\frac{F_2(x,Q^2)}{Q^2} = \frac{1}{4\pi^3}\int dr^2 \left\{ \left|\psi_{\rm T}(r,Q^2)\right|^2 + \left|\psi_{\rm L}(r,Q^2)\right|^2 \right\} \sigma_{\rm dp}(r^2) \tag{1.1}$$

where $\psi_{\rm T,L}$ are known functions that describe photon dissociation into a $q\bar{q}$ (dipole) pair. For massless quarks these functions have a property

$$\Phi_{\rm T,L}(u=rQ) = r^2 \left|\psi_{\rm T,L}(r,Q^2)\right|^2 , \tag{1.2}$$

i.e. $\Phi_{\rm T,L}$ depend only on a combined variable u. Dipole-proton cross-section $\sigma_{\rm dp}(r^2)$ has to be modeled. If

$$\sigma_{\rm dp}(r^2) = \sigma_0 f(r^2 Q_{\rm s}^2) \tag{1.3}$$

where f is dimensionless function (σ_0 sets the dimension) of dipole size r and momentum scale $Q_{\rm s}$ then

$$F_2(x,Q^2)/Q^2 = \text{function}\left(Q^2/Q_{\rm s}^2\right). \tag{1.4}$$

Here $\tau = Q^2/Q_{\rm s}^2$ is called *scaling variable* and $Q_{\rm s} = Q_{\rm s}(x)$ denotes the saturation momentum, which takes the following form

$$Q_{\rm s}^2 = Q_0^2\,(x/x_0)^{-\lambda} \tag{1.5}$$

that follows from the traveling wave solutions [3] of the nonlinear QCD evolution equations [4, 5] In what follows only the value of exponent λ will be of importance. Property (1.4) is called geometrical scaling [6].

In Fig. 1 we show that combined DIS data [7] indeed exhibit GS. Exponent λ has been extracted in a model-independent way in Ref. [8] and takes the value of 0.329.

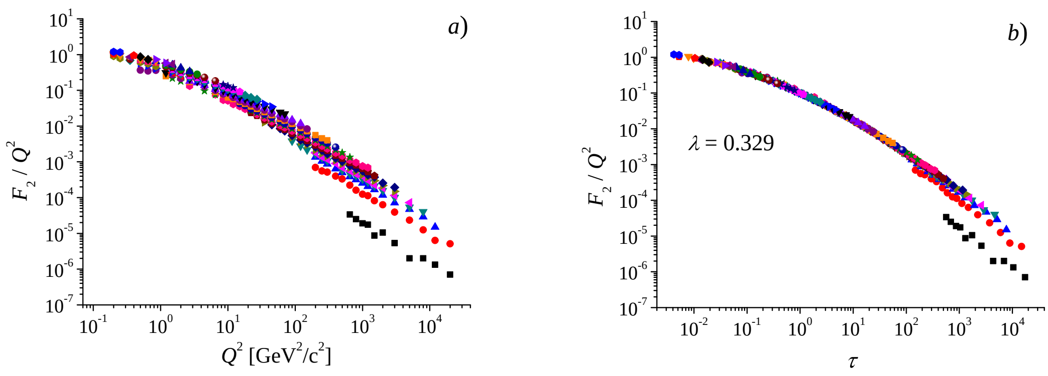

Figure 1: Left: $\gamma^* p$ cross-sections F_2/Q^2 as functions of Q^2 for fixed x. Different points correspond to different Bjorken x's. Right: the same but in function of scaling variable τ for $\lambda = 0.329$. Points in the right end of the plot correspond to large x's (due to kinematical correlation of the HERA phase space), and therefore show explicitly violation of geometrical scaling. (Figure from Ref. [8].)

2. Proton-proton scattering

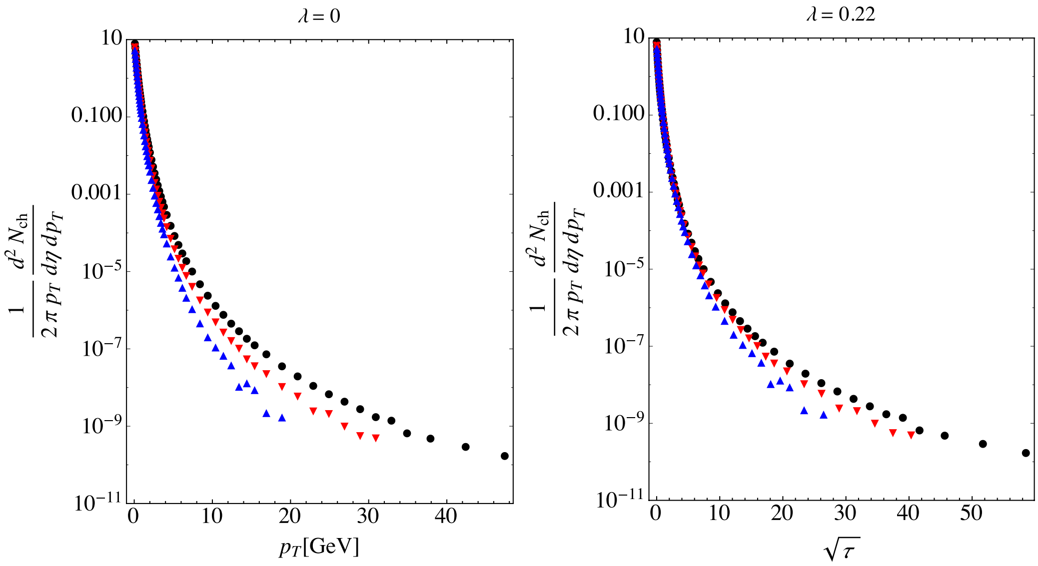

Figure 2: Data for pp scattering from ALICE [13] plotted in terms of p_T and $\sqrt{\tau}$. Full (black) circles correspond to $W = 7$ TeV, down (red) triangles to 2.76 TeV and up (blue) triangles to 0.9 TeV.

GS in DIS follows from the scaling property (1.3) of the dipole cross-section, which in turn is related to the unintegrated gluon distribution denoted in the following by $\varphi(k_T^2, x)$. Inclusive gluon cross-section can be expressed in terms of φ's in the k_T factorization scheme [9]:

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi}{2p_T^2}\int d^2\vec{k}_T\,\alpha_s(k_T^2)\varphi_1(x_1,\vec{k}_T^2)\varphi_2(x_2,(\vec{k}-\vec{p})_T^2). \tag{2.1}$$

Here $\varphi_{1,2}$ are unintegrated gluon densities and $x_{1,2}$ are gluon momenta fractions needed to produce a gluon of transverse momentum p_T and rapidity y:

$$x_{1,2} = e^{\pm y} p_T/\sqrt{s}. \tag{2.2}$$

Note that unintegrated gluon densities have dimension of area. This is at best seen from the very simple parametrization propsed by Kharzeev and Levin [10] or by Golec-Biernat and Wüsthoff [2] in the context of DIS:

$$\varphi_{KL}(k_T^2) = S_\perp \begin{cases} 1 & \text{for } k_T^2 < Q_s^2 \\ Q_s^2/k_T^2 & \text{for } k_T^2 < p_T^2 \end{cases} \quad \text{or} \quad \varphi_{GBW}(k_T^2) = S_\perp \frac{3}{4\pi^2}\frac{k_T^2}{Q_s^2}\exp\left(-k_T^2/Q_s^2\right). \tag{2.3}$$

Here $S_\perp$ is the transverse size given by inelastic cross-section (or its part) for the minimum bias inclusive multiplicity or in the case of DIS $S_\perp = \sigma_0$ is the dipole-proton cross-section for large dipoles. Another feature of the unintegrated glue (2.3) is the fact that φ depends on the ratio $k_T^2/Q_s^2(x)$ rather than on k_T^2 and x separately.

An immediate consequence of (2.1) is GS for the inelastic cross-section in mid-rapidity ($y \sim 0$)

$$\frac{d\sigma}{dyd^2p_{\rm T}} = S_\perp^2 \mathscr{F}(\tau) \qquad \text{or} \qquad \frac{1}{S_\perp}\frac{dN}{dyd^2p_{\rm T}} = \mathscr{F}(\tau) \tag{2.4}$$

where $\tau = p_{\rm T}^2/Q_{\rm s}^2(x)$ is scaling variable. If

$$d\sigma = S_\perp dN \tag{2.5}$$

then second of Eqs.(2.4) holds. This implies that particle spectra dN/dy at different energies should coincide if plotted in terms τ. In other words they should exhibit GS [11] (if we neglect logarithmic violations of GS due to $\alpha_{\rm s}$ and assume parton-hadron duality [12]). That this indeed happens is illustrated in Fig. 2. The best quality of GS is in this case achieved for $\lambda = 0.22$, which is different than λ extracted from DIS. In Ref. [14] we argue that this difference is removed if one assumes that the scaling observable is $d\sigma$ rather that dN, which implies that the proportionality factor in Eq. (2.5) is not energy independent $S_\perp$ but an inelastic cross-section $\sigma_{\rm in}(s)$.

Of course GS in pp is not perfect and extends only over the limited range up to $\sqrt{\tau} \sim 4$. Nevertheless it is still quite impressive, given the fact that strictly speaking GS is a property of produced gluons. Physical particles appear due to gluon fragmentation, they undergo final state interactions, and many of them are in fact produced from resonance decays. All these effects seem to preserve GS.

As a consequence of Eq. (2.4) both integrated multiplicity dN/dy and average transverse momentum $\langle p_{\rm T}\rangle$ grow as a power with energy [11]. This behavior is indeed seen in the data. Furthermore correlations of $\langle p_{\rm T}\rangle$ with $N_{\rm ch}$ are well described by GS supplemented with model calculations within Color Glass Condensate [15].

3. Heavy ion collisions

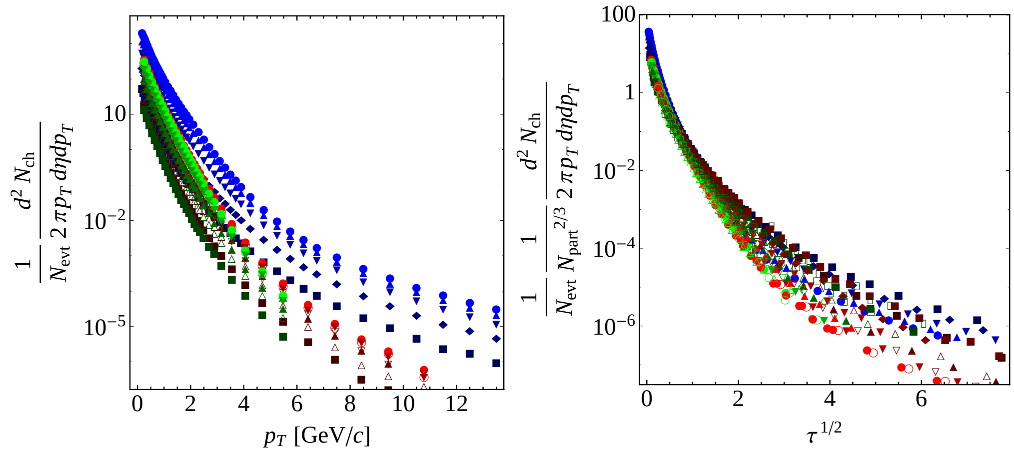

Figure 3: Illustration of geometrical scaling in heavy ion collisions at different energies and different centrality classes. Left panel shows charged particle distributions from ALICE [18], STAR [19, 20] and PHENIX [21, 22] plotted as functions of $p_{\rm T}$. In the right panel the same distributions are scaled according to Eq. (3.2).

While GS in pp scattering – as the property of the initial state – might have come as a surprise, it would be even more so if GS were present in heavy ion (HI) collisions. This is because strongly interacting matter undergoes hydrodynamical evolution before it finally hadronizes. Nevertheless, as we shall show below following Ref. [1], GS can be seen in particle spectra in HI collisions both for hadrons [16]. GS holds also and for photons [17] that, however, probe the initial stage of the collision.

HI data are divided into centrality classes that select events within certain range of impact parameter b. In this case both transverse area $S_\perp$ and the saturation scale Q_s^2 acquire additional dependence on centrality that is characterized by an average number of participants N_{part}. We have [17, 23]:

$$S_\perp \sim N_{\text{part}}^{2/3} \quad \text{and} \quad \mathrm{Q_s^2 \sim N_{part}^{1/3}}. \tag{3.1}$$

Therefore in HI collisions

$$\frac{1}{N_{\text{evt}}} \frac{dN_{\text{ch}}}{N_{\text{part}}^{2/3}\, d\eta d^2 p_{\text{T}}} = \frac{1}{Q_0^2} F(\tau) \;\; \text{where} \;\; \tau = \frac{p_{\text{T}}^2}{N_{\text{part}}^{1/3} Q_0^2} \left(\frac{p_{\text{T}}}{W}\right)^{\lambda}. \tag{3.2}$$

Note that by selecting certain centrality class we in fact select an overlap $S_\perp$ between interacting ions and therefore one can safely use relation (2.5).

In Fig. 3 we plot LHC and RHIC data in terms of p_{T} (left panel) and $\sqrt{\tau}$ for $\lambda = 0.3$ (right panel). One can see an approximate scaling of, however, worse quality than in the pp case.

4. Summary

A wealth of data in ep and in hadronic collisions exhibits GS. In this note we have only mentioned some examples. The most important topics not included here are extension of GS to the case of identified particles [24] and GS violation for $y \neq 0$ [25].

GS may be interpreted as a signature of saturation. However, one has to bare in mind that it is a linear part of QCD evolution equations that develops GS. Nonlinearities serve as a "damping" that prevents scattering amplitudes from growing over the unitarity limit and – at the same time – making the entire solution to take asymptotically the scaled form.

Many aspects of geometrical scaling require further studies. Firstly, new data from the LHC run II (to come) have to be examined. On theoretical side the universal shape $\mathscr{F}(\tau)$ has to be found and its connection to the unintegrated gluon distribution has to be studied. That will finally lead to probably the most difficult part, namely to the breaking of GS in pp and in HI.

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