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TIME AND PHYSICAL GEOMETRY
A formalization of Putnam's proof

Abstract. Putnam's proof that time flow is incompatible with Relativity is underestimated, mostly due to Stein's interpretation of the notion of reality in it as a two-term relation. This interpretation makes it vulnerable to easy criticism and makes various ways of escaping its conclusion possible. An alternative approach is proposed, resulting in a formalization which seems closer to Putnam's intentions where reality is interpreted as a non-relational property. Although it makes the proof immune to all standard strategies of blocking the proof, it reveals its real weak point which consists in assuming an overly strong interpretation of the principle of relativity.

Keywords: time flow, eternalism, relativity, simultaneity, spacetime

1. Introduction

Hilary Putnam's Relativity-based proof [14] that time does not flow[1] is commonly regarded as inconclusive. I subscribe to this opinion in principle, but for different reasons. It is usual to rely on the alleged correction of Putnam's reasoning by Howard Stein [19], according to which Putnam should have understood the notion *real* as represented by the relation $\mathbb{R}$ that he spoke about. However, Putnam has explicitly refused to accept this correction[2] as it would make his reasoning indefensible. My aim here is to attempt to formalize the original proof without any such allegedly

[1] Strictly speaking: that future events are as real as the present ones. However, this would exclude time flow, at least in its most natural understanding.

[2] "Stein's objection to my argument was that I overlooked the possibility of *relativizing the notion of reality* [...]. In my view, Stein simply misses the issue I was addressing, which is whether future events are real in the standard metaphysical

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charitable interpretations. It is hoped that this makes it safe from the standard criticisms whilst revealing its true weak point.

At the outset I intend to formalize the first part of Putnam's reasoning, where he proves the incompatibility of Special Relativity with the assumption that only present *things*[3] are real. Then, I will formalize the second part, where he proves that, even if this assumption is weakened in a reasonable way that avoids inconsistency, in the light of Special Relativity, it enables one to prove that all things are real, no less than the present ones. Next, I will show that this second part may be easily generalized to also hold in the framework of General Relativity. Finally, I will discuss various strategies for blocking the proof, with the intention of showing that only the one I have chosen is successful.

2. The first proof

Strictly speaking, Putnam's paper contains the proofs of two different theorems. The first one is the inconsistency of the "man on the street's" view of the nature of time with Special Relativity (SR). The first step is taking the assumption that, according to Putnam, expresses this view:

(1) All (and only) things that exist *now* are real.[4] [14, p. 240]

which may be formalized as follows [4, p. 46]:

(1) $\forall y[n(y) \Leftrightarrow r(y)]$,

where '$n(y)$' stands for 'y exists *now*', '$r(y)$' for 'y is real' and the variable 'y' takes values from the set of events, since it follows from the context that by *things* Putnam means events.

Then, two further assumptions follow which characterize the concept *real* as it appears in (1). The first one states:

I. I-now am real. [14, p. 240]

which may be formalized by [4, p. 46]:

I. $e \in I \ \wedge \ n(e) \ \wedge \ r(e)$,

understanding of "*real*", on which what is "real" is precisely supposed to be *mind-and-observer-independent*." [15]

[3] We will see that Putnam understood 'things' as including 'events'.

[4] Thus, the view that Putnam ascribes to the "man on the street" is just *presentism* [see 10, p. 123].

where I is *my* worldline and e is the event belonging to this worldline that represents *me-now*. Putnam remarks: "Of course, this assumption changes each time I announce that I am making it, since 'I-now' refers to a different instantaneous 'me' "; in other words, it is indexical. What is more, the predicate 'exists now' is also indexical. To avoid this complication, one may try to substitute this assumption with its logical consequence:

Somebody is real

to be formalized by:

$$\exists\alpha\exists x[x \in \alpha \ \wedge \ r(x)],$$

where the variable 'α ' takes values from the class of possible worldlines of an observer and the variable 'x' takes values from the set of events. However, we will see that a still weaker assumption suffices:

Something is real

to be formalized by:

I*. $\exists x\, r(x)$.

In turn, the second assumption states:

> II. At least one other observer is real, and it is possible for this other observer to be in motion relative to me. [14, p. 240]

which may be formalized by [4, p. 47]:

II. $\exists\beta\exists y[y \in \beta \ \wedge \ r(y) \ \wedge \ \Diamond M(\beta, I)]$,

where the variable 'β' takes values from the class of possible worldlines of this other observer, and M is the relation of the first term being in motion relative to the second. The only reasonable way of eliminating the indexical term 'me' from it seems to be by strengthening this assumption to:

For any observer, at least one other observer is real, and it is possible for this other observer to be in motion relative to the former

to be formalized by:

II*. $\forall\alpha\exists\beta\exists y[y \in \beta \ \wedge \ r(y) \ \wedge \ \Diamond M(\beta, \alpha)]$.

The following assumption, called by Putnam "the principle that There Are No Privileged Observers", arises:

> III. If it is the case that all and only the things that stand in a certain relation R to me-now are real, and you-now are also real, then it is also the case that all and only the things that stand in the relation R to you-now are real. [14, p. 241]

which may be formalized by [4, p. 47]:

$$\text{III. } \forall y[R(y,e) \Leftrightarrow r(y)] \ \wedge \ e' \in Y \ \wedge \ r(e') \ \Rightarrow \ \forall z[R(z,e') \Leftrightarrow r(z)],$$

or equivalently:

$$\forall y[R(y,e) \Leftrightarrow r(y)] \ \Rightarrow \ \forall z\big(e' \in Y \ \wedge \ r(e') \ \Rightarrow \ [R(z,e') \Leftrightarrow r(z)]\big),$$

where e' represents *you-now* in the spacetime, Y is *your* worldline and R is some physical[5] relation, "independent of the choice of a coordinate system [...] and to be definable in a "tenseless" way in terms of the fundamental notions of physics", chosen independently of "anything accidental". Moreover, in the light of assumption II, *you* may be in motion relative to *me*.

Putnam notes that, if R is taken to be simultaneity and classical[6] physics is assumed, then, on the view (1), "all and only the things that stand in the relation R to me-now are real, and the principle III is satisfied because the relation of simultaneity is transitive". Unfortunately, no link is seen between (1) and such alleged consequence of it. It seems that he must have implicitly made one more assumption, which might read as follows:

> *All and only the things that stand in a certain relation R to me-now exist now*,

where the relation R is assumed to be the same as in III, its formalization would be [4, p. 48]:

$$\text{(N) } \forall y[R(y,e) \Leftrightarrow n(y)]$$

and it would be natural to choose simultaneity as this relation. This enables us to derive from (1):

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[5] I.e., it seems, operationally definable.

[6] I.e. here: non-relativistic.

All and only the things that stand in a certain relation R to me-now are real,

which may be formalized by [4, p. 48]:

(1′) $\forall y[R(y,e) \Leftrightarrow r(y)]$.

Instead of taking this auxiliary assumption, Putnam could simply reinterpret (1) as (1′). This enables us to avoid the indexical expression 'exist now'. However, another indexical expression still remains, namely 'me-now'. In addition, the term 'you-now' is also indexical. Fortunately, we can eliminate them both as well since the denotation of constants 'e′' and 'Y' is unspecified, they may be substituted for by variables. Moreover, as assumption III establishes the equivalence of all observers, it may be strengthened to:

If, for some observer, it is the case that all and only the things that stand in a certain relation R to this observer-now are real, then, for any other observer-now that is also real, it is also the case that all and only the things that stand in the relation R to that other observer-now are real,

which may be formalized by:

III*. $\exists\alpha\exists x\forall y\big(x \in \alpha \ \wedge\ [R(y,x) \Leftrightarrow r(y)]\big) \ \Rightarrow\ \forall\beta\forall y\forall z\big(y \in \beta \ \wedge\ r(y) \ \Rightarrow\ [R(z,y) \Leftrightarrow r(z)]\big)$,

whereas (1′) may be weakened to:

For some observer, it is the case that all and only the things that stand in a certain relation R to this observer-now are real,

which may be formalized by:

(1′*) $\exists\alpha\exists x\forall y\big(x \in \alpha \ \wedge\ [R(y,x) \Leftrightarrow r(y)]\big)$.

From (1′*) and III*, as well as from (1′) and III, we obtain the following theorem:

For any observer, if this observer-now is real, then all and only the things that stand in the relation R to this observer-now are real,

to be formalized by [4, p. 48]:

$$\forall\beta\forall y\forall z\big(y \in \beta \ \wedge\ r(y) \ \Rightarrow\ [R(z,y) \Leftrightarrow r(z)]\big).$$

In classical (i.e., non-relativistic) physics this does not result in any trouble, since then, as Putnam notes,

> if we [...] take the relation R to be the relation of simultaneity, then, on the view (1) [understood as our (1′)], it is true that all and only the things that stand in the relation R to me-now are real, and the principle III is satisfied because the relation of simultaneity is transitive.
>
> [14, p. 241]

Now, in the framework of Special Relativity, we must take into account the relativity of simultaneity. This means that, instead of one absolute simultaneity, there is a class of relative simultaneities defined in different reference frames. Equivalently, one may say that the simultaneity which was a two-term relation becomes a three-term relation, the third term being some object that represents a reference frame [6]. We may choose the worldline of a possible observer resting in the frame in question for this role. If so, assumption (1′*) must be corrected to:

> *For some observer, it is the case that all and only the things that stand in a certain relation R to this observer-now in this observer's frame are real*,

which may be formalized by [cf. 4, p. 48]:

$$(1''^*)\ \ \exists\alpha\exists x\forall y\big(x \in \alpha\ \wedge\ [R(y,x,\alpha) \Leftrightarrow r(y)]\big),$$

whereas III* must be corrected to:

> *If, for some observer, it is the case that all and only the things that stand in a certain relation R to this observer-now in this observer's frame are real, and other observer-now is also real, then it is also the case that all and only the things that stand in the relation R to that other observer-now in that observer's frame are real*,

to be formalized by [cf. 4, p. 49]:

$$\text{III}'^*.\ \ \exists\alpha\exists x\forall y\big(x \in \alpha\ \wedge\ [R(y,x,\alpha) \Leftrightarrow r(y)]\big)\ \Rightarrow\ \forall\beta\forall y\forall z\big(y \in \beta\ \wedge\ r(y)\ \Rightarrow\ [R(z,y,\beta) \Leftrightarrow r(z)]\big).$$

It is easy to see that assumptions (1″*) and III′* enable us to obtain the following lemma:

For any observer, if this observer-now is real, then all and only the things that stand in the relation R to this observer-now in this observer's frame are real,

which may be formalized by [cf. 4, p. 49]:

(L*) $\forall\beta\forall y\forall z\big(y \in \beta \ \wedge \ r(y) \ \Rightarrow \ [R(z, y, \beta) \Leftrightarrow r(z)]\big)$.

On the other hand, in a world where Special Relativity holds, it is a fact that, if I-now am in it, you-now are simultaneous with me-now in my reference frame and you move relative to me, then some event is simultaneous with you-now in your frame, but is not simultaneous with me-now in my frame. In particular, this is the case if "the following (perfectly possible) physical situation" is actual: "you-now and I-now are at the same place now, but moving with relative velocities which are very large" [14, p. 242]. In such a situation, the following condition is satisfied:

$$\exists z[R(e, e, I) \ \wedge \ R(z, e, Y) \wedge \ \neq R(z, e, I)],$$

or, slightly more generally [4, p. 49]:

(FP1) $\exists z[R(e', e, I) \ \wedge \ R(z, e', Y) \ \wedge \ \neg R(z, e, I)]$,

whereas it is assumed that $e' \in Y$, so:

$$\exists z[R(e', e, I) \ \wedge \ e' \in Y \ \wedge \ R(z, e', Y) \ \wedge \ \neg R(z, e, I)],$$

where the event that satisfies it may be a future one. Still more generally, a weaker condition is satisfied [4, p. 49]:

(F1) $\exists z\exists\beta\exists y[R(y, e, I) \ \wedge \ R(z, y, \beta) \ \wedge \ \neg R(z, e, I)]$,

or better:

$$\exists z\exists\beta\exists y[R(y, e, I) \ \wedge \ y \in \beta \ \wedge \ R(z, y, \beta) \ \wedge \ \neg R(z, e, I)],$$

which, however, may be generalized to:

(F1*) $\forall\alpha\forall x\exists z\exists\beta\exists y[x \in \alpha \wedge R(y, x, \alpha) \wedge y \in \beta \wedge R(z, y, \beta) \wedge \neg R(z, x, \alpha)]$,

since it holds generally, at any time and no matter, in which reference frame I am at rest.

Putnam observes that, as a result, "*future* things (or events) are already real", because, if "these things stand in the relation R to you-now, and you-now are real", then "the principle III requires that I call

these future things and events real". However, he notices immediately: "But, actually, I now have a contradiction: for these future things do not stand in the relation R to me-now, and so my assumption that all and only the things that stand in *this* relation R to me-now are real was already inconsistent with the principle that There Are No Privileged Observers" [14, p. 242]. Let us obtain this conclusion in a formal way. Thanks to the lemma (L*), which is derivable from (1″*) and III′*, it is easy to show that assumptions I*, (1″*) and III′*, together with (F1*), enable us to derive [4, p. 50]:

$$\exists z[r(z) \ \wedge \ \neg r(z)],$$

i.e., a manifest contradiction.

3. The second proof

Unfortunately, the latter conclusion undermines the former, making the proof of it trivial. Putnam remarks:

> The difficulty is obvious: what the principle that There Are No Privileged Observers requires is simply that the relation R be transitive [...]. Simultaneity-in-my-coordinate-system has this property [...]; but [...] is not admissible as a choice of R, because it depends on the coordinate system. And the relation "x is simultaneous with y in the coordinate system of x" [...], while admissible, is not transitive.
>
> [14, pp. 242–243]

But what does the expression 'the coordinate system of x' mean? If x is supposed to be somebody-now, i.e., an event on somebody's worldline, then the expression is ambiguous, since the same event may belong to many different worldlines of possible observers. To avoid this ambiguity, one would have to specify the worldline α the event x is intended to belong to.

Next, let us try to understand what it means to say that the relation of the simultaneity of something with somebody-now in this somebody's coordinate system might be, but, as a matter of fact, is not, transitive. It seems that, in our formalization, it must mean at least:

$$\forall x\forall y\forall z\big(\exists\alpha\exists\beta[y \in \beta \wedge R(z,y,\beta) \wedge x \in \alpha \wedge R(y,x,\alpha)] \Rightarrow \exists\gamma[x \in \gamma \wedge R(z,x,\gamma)]\big).$$

It is clear that, in Special Relativity, the relation of relative simultaneity does not have this property.

To avoid the contradiction to which our assumptions led, it seems necessary to weaken at least one of them. Putnam regards as desirable weakening assumption III and adopting, instead of (1),

> the following principle, which is one-half of (1):
> (2) All things that exist now are real. [14, p. 243]

to be formalized by [4, p. 50]:

(2) $\forall x[n(x) \Rightarrow r(x)]$,

which he intends now to mean that "all things that exist now according to my coordinate system are real" [4, p. 50]:

(2″) $\forall y[R(y, x, \alpha) \Rightarrow r(y)]$,

i.e., in the present approach:

> For some observer, it is the case that all things that stand in a certain relation R to this observer-now in this observer's frame are real.

This is just "one half of" our (1″*), to be formalized by:

(2″*) $\exists\alpha\exists x\forall y(x \in \alpha \wedge [R(y, x, \alpha) \Rightarrow r(y)])$.

Consequently, assumption III must be expressed by:

> *If, for some observer, it is the case that all things that stand in a certain relation R to this observer-now in this observer's frame are real, and the other observer-now is also real, then it is also the case that all things that stand in the relation R to that other observer-now in that observer's frame are real*,

to be formalized by:

$\exists\alpha\exists x\forall y\big(x \in \alpha \wedge [R(y, x, \alpha) \Rightarrow r(y)]\big) \Rightarrow \forall\beta\forall y\forall z\big(y \in \beta \wedge r(y) \Rightarrow [R(z, y, \beta) \Rightarrow r(z)]\big)$,

or equivalently [cf. 4, p. 51]:

III″*. $\exists\alpha\exists x\forall y\big(x \in \alpha \wedge [R(y, x, \alpha) \Rightarrow r(y)]\big) \Rightarrow \forall\beta\forall y\forall z\big(y \in \beta \wedge R(z, y, \beta) \Rightarrow [r(y) \Rightarrow r(z)]\big)$.

Now, assumptions (2″*) and III″* enable us to obtain the following lemma:

For any observer, if this observer-now is real, then all things that stand in the relation R to this observer-now in this observer's frame are real,

which may be formalized by [cf. 4, p. 51]:

(L′*) $\forall\beta\forall y\forall z\big(y \in \beta \ \wedge \ R(z, y, \beta) \ \Rightarrow \ [r(y) \ \Rightarrow \ r(z)]\big)$.

Instead of the contradiction, on the basis of (F1*), we now only get:

$$\exists\alpha\exists x\exists z[r(x) \ \wedge \ r(z) \ \wedge \ \neg R(z, x, \alpha)],$$

i.e., for some observer, who is real, an event must also be real which is not simultaneous with this observer in his frame. In particular, some such event must be a future one for this observer. In other words, as Putnam observes, "we quickly see that future things must be real" [14, p. 243].

This result may look rather modest, since we still cannot prove that all things (past, present and future) are real. However, as Putnam observes, "the argument can obviously be extended [...] to show that *all* future things are real ("things" here includes "events"), and likewise that all *past* things are real". It is enough to regard as real "every thing and event which bears the *transitive closure* of R to me (i.e., which bears R to me, or which bears R to something that bears R to me, [...] or [...])" [14, p. 243]. This is justified by the following observation: in Special Relativity, if R is relative simultaneity, as above, it is a fact that [4, p. 52]:

(F2) $\forall z\exists\beta\exists y[R(y, e, I) \ \wedge \ R(z, y, \beta)]$,

or better:

$$\forall z\exists\beta\exists y[R(y, e, I) \ \wedge \ y \in \beta \ \wedge \ R(z, y, \beta)],$$

which may be generalized to:

(F2*) $\forall\alpha\forall x\forall z\exists\beta\exists y[x \in \alpha \ \wedge \ R(y, x, \alpha) \ \wedge \ y \in \beta \ \wedge \ R(z, y, \beta)]$.

Thanks to lemma (L′*), derivable from (2″*) and III″*, it is easy to show that the assumptions I*, (2″*) and III″*, together with (F2*), enable us to derive (cf. [4, p. 52]):

(E) $\forall z\, r(z)$,

which expresses the thesis of *eternalism* [see 10, p. 122]:

All things are (equally) real.

This means that time does not flow, if the *flow* of time is understood as consisting in temporal *becoming* real, which is absent if nothing *becomes* real. One may object that the flow, or the *passage* of time, may be understood in a different way, compatible with eternalism, but it is rather unclear what it would then consist in. For instance, Tim Maudlin is of such an opinion, but he does not explain in what way, according to his view, "the passage of time connotes more than just an intrinsic asymmetry" [13, p. 109]. Similarly, in the case of the so-called *moving spotlight* view, which introduces the primitive monadic property of *presentness*, the nature of which is left unexplained, is rather questionable [18, p. 260]. On the other hand, the aim of this paper is not to defend the interpretation of Putnam's reasoning as a proof that time does not flow, but only to examine his own two proofs: the first against presentism and the second for eternalism. Thus, an extensive discussion of other ways of understanding the flow of time here, especially the ones that assume just from the start what Putnam intended to prove, would be superfluous.

From (E) it follows trivially that [4, p. 52]:

(E′) $\forall z[z \in I \ \Rightarrow \ r(z)]$,

i.e., that all events of my life are real — including future ones! Thus, although SR does not contradict the weakened formulation (2) of the "man on the street" assumption, it enables a consequence that is clearly inconsistent with his view about time to be derived from this assumption. It is worth noting that, for this, the transitivity of R is not necessary, since no assumption here requires this property.

4. Generalization

One might object that condition (F2*) holds generally only in SR, which is merely a local approximation of General Relativity (GR) as a theory that is true about our world. Unfortunately, (F2) is a special case of a weaker condition [4, p. 53]:

(F3) $\forall z \exists n \exists \beta_1 \ldots \exists \beta_n \exists y_1 \ldots \exists y_n [R(y_1, e, I) \ \wedge \ R(y_2, y_1, \beta_1) \ \wedge \cdots \wedge R(z, y_n, \beta_n)]$,

which means that just z "bears the transitive closure of R to me", as Putnam would have said. It is easy to see that (F3) holds also in GR, since the events may be always so chosen that the events of the successive

pairs in this chain are sufficiently close to each other that, for this pair, SR (approximately) holds. A generalization is straightforward:

(F3*) $\forall\alpha\forall x\forall z\exists n\exists\beta_1\ldots\exists\beta_n\exists y_1\ldots\exists y_n[x\in\alpha\ \wedge\ R(y_1,x,\alpha)\ \wedge\ y_1\in\beta_1\wedge R(y_2,y_1,\beta_1)\ \wedge\ \cdots\ \wedge\ y_n\in\beta_n\ \wedge\ R(z,y_n,\beta_n)]$.

It may be shown that, together with I*, (2″*) and III″*, it enables deriving (E). Thus, a recourse to GR is of no help.

Another way of trying to block the proof might be questioning simultaneity in the role of R. However, it is not easy to find a relatively natural candidate to this role that would not satisfy (F3*). Moreover, let us observe that from I it follows that:

$$\forall x[R(x,e,I)\ \Rightarrow\ r(x)]\ \Leftrightarrow\ \forall x\big(R(x,e,I)\ \Rightarrow\ [r(e)\Leftrightarrow r(x)]\big).$$

This suggests strengthening III″* to [cf. 4, p. 53]:

III‴*. $\exists\alpha\exists x\forall y\big(x\in\alpha\wedge[R(y,x,\alpha)\ \Rightarrow\ r(y)]\big)\ \Rightarrow\ \forall\beta\forall y\forall z\big(y\in\beta\ \wedge\ R(z,y,\beta)\ \Rightarrow\ [r(y)\Leftrightarrow r(z)]\big)$.

From (2″*) and III‴* it follows that:

(L″*) $\forall\beta\forall y\forall z\big(y\in\beta\ \wedge\ R(z,y,\beta)\ \Rightarrow\ [r(y)\Leftrightarrow r(z)]\big)$.

On the other hand, in SR, apart from (F2), it holds that [4, p. 53]:

(F2′) $\forall z\exists\beta\exists y\big(R(y,e,I)\ \wedge\ [R(z,y,\beta)\ \vee\ R(y,z,\beta)]\big)$,

or better:

$$\forall z\exists\beta\exists y\big(R(y,e,I)\ \wedge\ ([y\in\beta\ \wedge\ R(z,y,\beta)]\ \vee\ [z\in\beta\ \wedge\ R(y,z,\beta)])\big),$$

which may be generalized to:

(F2′*) $\forall\alpha\forall x\forall z\exists\beta\exists y\big(x\in\alpha\wedge R(y,x,\alpha)\wedge([y\in\beta\wedge R(z,y,\beta)]\ \vee\ [z\in\beta\ \wedge\ R(y,z,\beta)])\big)$.

Together with I* and (L″*), it enables us to derive (E). Indeed, we obtain (a sketch — some trivial intermediate steps are omitted) [cf. [3, p. 99]:

1.	$\forall\beta\forall y\forall z\big(y\in\beta\ \wedge\ R(z,y,\beta)\ \Rightarrow\ [r(y)\Leftrightarrow r(z)]\big)$	(L″*)
2.	$\forall\beta\forall y\forall z\big(y\in\beta\ \wedge\ R(z,y,\beta)\ \Rightarrow\ [r(y)\ \Rightarrow\ r(z)]\big)$	1
3.	$x\in\alpha\ \wedge\ R(y,x,\alpha)\ \Rightarrow\ [r(x)\ \Rightarrow\ r(y)]$	2
4.	$y\in\beta\ \wedge\ R(z,y,\beta)\ \Rightarrow\ [r(y)\ \Rightarrow\ r(z)]$	2
5.	$\forall\beta\forall y\forall z\big(y\in\beta\ \wedge\ R(z,y,\beta)\ \Rightarrow\ [r(z)\ \Rightarrow\ r(y)]\big)$	1
6.	$z\in\beta\ \wedge\ R(y,z,\beta)\ \Rightarrow\ [r(y)\ \Rightarrow\ r(z)]$	5

7. $\big([y \in \beta \wedge R(z,y,\beta)] \vee [z \in \beta \wedge R(y,z,\beta)]\big) \Rightarrow [r(y) \Rightarrow r(z)]$ 4, 6
8. $\big(x \in \alpha \wedge R(y,x,\alpha) \wedge ([y \in \beta \wedge R(z,y,\beta)] \vee [z \in \beta \wedge R(y,z,\beta)])\big) \Rightarrow \big([r(x) \Rightarrow r(y)] \wedge [r(y) \Rightarrow r(z)]\big)$ 3, 7
9. $\big(x \in \alpha \wedge R(y,x,\alpha) \wedge ([y \in \beta \wedge R(z,y,\beta)] \vee [z \in \beta \wedge R(y,z,\beta)])\big) \Rightarrow [r(x) \Rightarrow r(z)]$ 8
10. $\exists\beta\exists y\big(x \in \alpha \wedge R(y,x,\alpha) \wedge ([y \in \beta \wedge R(z,y,\beta)] \vee [z \in \beta \wedge R(y,z,\beta)])\big) \Rightarrow [r(x) \Rightarrow r(z)]$ 9
11. $\forall\alpha\forall x\forall z\exists\beta\exists y\big(x \in \alpha \wedge R(y,x,\alpha) \wedge ([y \in \beta \wedge R(z,y,\beta)] \vee [z \in \beta \wedge R(y,z,\beta)])\big) \Rightarrow [r(x) \Rightarrow r(z)]$ 10
12. $\forall\alpha\forall x\forall z\exists\beta\exists y\big(x \in \alpha \wedge R(y,x,\alpha) \wedge ([y \in \beta \wedge R(z,y,\beta)] \vee [z \in \beta \wedge R(y,z,\beta)])\big)$ (F2′*)
13. $r(x) \Rightarrow r(z)$ 11, 12
14. $\exists y\, r(x) \Rightarrow \forall z\, r(z)$ 13
15. $\exists x\, r(x)$ I*
16. $\forall z\, r(z)$ 14, 15

Furthermore, relative simultaneity satisfies a weaker condition [cf. 4, p. 53]:

$$\forall z\exists n\exists\beta_1 \ldots \exists\beta_n\exists_1 \ldots \exists y_n\big(R(y_1,e,I) \wedge ([y_1 \in \beta_1 \wedge R(y_2,y_1,\beta_1)] \vee [y_2 \in \beta_1 \wedge R(y_1,y_2,\beta_1)]) \wedge \cdots \wedge ([y_n \in \beta_n \wedge R(z,y_n,\beta_n)] \vee [z \in \beta_n \wedge R(y_n,z,\beta_n)]\big)$$

and its generalization:

(F3′*) $\forall\alpha\forall x\forall z\exists n\exists\beta_1 \ldots \exists\beta_n\exists y_1 \ldots \exists y_n\big(x \in \alpha \wedge R(y_1,x,\alpha) \wedge ([y_1 \in \beta_1 \wedge R(y_2,y1,\beta_1)] \vee [y_2 \in \beta_1 \wedge R(y_1,y_2,\beta_1)]) \wedge \cdots \wedge ([y_n \in \beta_n \wedge R(z,y_n,\beta_n)] \vee [z \in \beta_n \wedge R(y_n,z,\beta_n)])\big)$.

It is easy to see that it is also a weakened version of (F3*), so it is satisfied in GR, as well. Unfortunately, together with I* and (L″*), it yields (E).

5. Standard strategies for blocking the proof

Since in GR locally defined simultaneity satisfies (F2′*), leaving the framework of SR in favor of this more realistic theory will not suffice in terms of blocking the proof. A more promising strategy might seem to be questioning relative simultaneity as a criterion of reality. A natural alternative might seem to be the adoption of some invariant relation. It may still be regarded as a three-term relation, but with the third term

being redundant. Relatively serious candidates might be: spatial separation (or "topological simultaneity"), belonging to the chronological, causal or lightlike (or horismoidal) past of somebody-now [2], belonging to his "Alexandrov present" [17], or even spatiotemporal coincidence.

The last choice, which corresponds to the idea of strict locality of becoming [7], does not seem a reasonable proposal. Assuming that the view of the "man on the street" is expressed by (1″*) then implies solipsism. On the other hand, although assuming that it is expressed only by (2″*) does not imply solipsism, it does not provide any means for determining which events, apart from me-now, are also real. Thus, if one wants to avoid both solipsism and the possibility that some future events are also real, then another relation seems preferable. Unfortunately, all the remaining satisfy (F3′*).

Another strategy might consist in neutralizing (F3′*) by questioning strengthening III″* to III‴*, which might really raise doubts. This means imposing on R the stronger condition (F3*). It is easy to see that it is satisfied by both the spatial separation and the "Alexandrov present". On the other hand, as R is intended as a criterion of presence, the relations of causal or chronological precedence must be excluded. What remains is at most lightlike precedence, which defines the "visual present" of any observer-now, containing all the events that, in principle, may be directly seen by this local observer. This corresponds to the "backward null cone definition" of simultaneity [16].

Although this relation does not satisfy (F3*) to the full extent, it satisfies it with respect to events belonging to the causal past of a given observer-now, since causal preceding is just the transitive closure of a lightlike preceding. In particular, the causal past of a given event contains all the events that are chronologically earlier than it. Consequently, the choice of lightlike precedence as R is not compatible with the presentist version of *transientism* [8], but rather with its more liberal version, known as *Growing Block Theory* (GBT) [9], according to which past events are as real as present ones. It is worth noting that GBT is compatible only with the "man on the street's" view as expressed by (2″*), but not as expressed by (1″*).

Next, one might ask if there is such an event (as possible observer-now) that all and only the events that belong to its causal past are real. It is clear that a positive answer to this question would mean a drastic breaking "the principle that There Are No Privileged Observers". On the other hand, a negative answer would mean that for any possible

observer-now, there are events that are real, although they do not bear the transitive closure of lightlike precedence to him. Thus, the choice of this relation as R does not exclude the possibility that all events are real, after all. Anyway, another choice of R seems preferable.

Let us, then, consider relative, i.e. non-invariant, relations. As we have already seen, relative simultaneity satisfies (F3*). *A fortiori*, this condition is satisfied by the relation of being either relatively simultaneous or slightly earlier, defining the so-called *specious present* [11] of a given observer-now. It is easy to see that its transitive closure is the relation of being either relatively simultaneous or relatively earlier, so its choice would be incompatible with presentism even in the framework of non-relativistic physics, although it is compatible with GBT. There seems to be no other natural relative relation as a candidate for R.

6. A non-standard strategy for blocking the proof

It seems that all the standard strategies of blocking the proof, consisting in choosing as R some relation other than relative simultaneity, fail. In the proof, it is important that the variables α and β take values from the set of worldlines of possible observers in different states of motion. Thus, an alternative strategy might consist in restricting their scope.

To some extent, some such restriction may be justified in the framework of GR. In this framework, a clearly privileged status is possessed by inertial observers, whose worldlines are fragments of timelike geodesics. Unfortunately, such a restriction does not block the proof, and the standard interpretation of this theory does not enable any further restriction.

However, this situation changes once one chooses the non-standard, Lorentzian interpretation of Relativity [12, 1]. According to it, although all the inertial observers are physically equivalent, some of them are privileged on a more fundamental, ontological level. Such a status may be attributed to observers locally resting relative to the ether. In the framework of SR, it is natural to assume that the ether rests in a global inertial frame, but the lack of such a global frame in GR is no serious obstacle. What is essential is that a global relation of simultaneity, coinciding locally with relative simultaneities in the local ether frames, is definable.

Such a privileged status cannot be attributed to local frames moving relative to the ether, since in them physical phenomena are deformed as

a result of their motion relative to the appropriate local ether frames. In particular, it is true for any physical synchronizing procedure defining relative simultaneity in each of them. After imposing the appropriate restriction on the range of the variables α and $\beta_1, \ldots, \beta_n$, relative simultaneity does not satisfy even the weakest condition (F3′*), let alone the stronger conditions (F2'*) or (F2*). Consequently, the proof is blocked even after the controversial strengthening of III″* to III‴*. What is more, condition (F1*) is not satisfied either, which means that the first proof is also impossible, in spite of strengthening (2″*) to (1″*).

Let us assume now that an observer is resting in a local ether frame — which he cannot know as a result of the principle of relativity — and the above restriction is imposed on the variables, but the role of R, instead of simultaneity, is played by the relation belonging to the relative specious present of this observer-now in his frame. It is clear that even then this relation satisfies (F3′*), and even (F3*), if restricted to his relative past. As a result, if this observer-now is real, then all events in his relative past are also real. This means that if such a choice of R is compatible with time flow, then only with its GBT, and not the presentist model. What is more, (F1*) is also satisfied, so it is compatible only with (2*), and not with (1*). This would be true also in the framework of non-relativistic physics. Thus, condition (1*) is specific for the presentist version of transientism, whereas (2*) is compatible also with its GBT version.

7. Summary and conclusions

It transpires that the success of Putnam's proofs depends fundamentally on the choice of the standard geometrical interpretation of Relativity. It is extremely difficult, if at all possible, to block them in the framework of this interpretation. On the other hand, in the framework of the dynamical interpretation, not only is the second proof impossible but the first one is too, which is based on stronger assumptions. Consequently, the opinion that they rest on Relativity unconditionally is unjustified. Thus, if one is interested in reconciling the acceptance of this theory with the intuition that time flows, the choice of the dynamical interpretation seems inescapable. Independent, strong theoretical reasons speak for this as well [5].

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