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# Achievements and challenges in understanding nucleon-deuteron reactions

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**Summary.** — Results on three-nucleon (3N) elastic scattering below the pion production threshold are discussed with an emphasis on a need for a three-nucleon force (3NF). The large discrepancies found between a theory based on numerical solutions of 3N Faddeev equations with (semi)phenomenological nucleon-nucleon (NN) potentials only and data point to the need for 3NF's. This notion is supported by the fact that another possible reason for the discrepancies in elastic nucleon-deuteron (Nd) scattering, relativistic effects, turned out to be small. Results based on a new generation of chiral NN forces (up to $N^4$LO) alone or combined with $N^2$LO 3NF support predictions found with standard interactions. To resolve higher energy discrepancies found in nucleon-deuteron (Nd) reactions requires application of a chiral 3NF up to at least $N^3$LO order of chiral expansion.

## 1. – Introduction

In the 3N system for the first time 3NF's come into play making it a valuable source of information on 3NF properties and their significance in the nuclear Hamiltonian.

The need for 3NF's was established when three- and four-nucleon bound states have been solved exactly [1, 2] using high precision NN potentials which describe the NN data set with high precision ($\chi^2$/datum $\approx 1$) [3-5]. The observed underbinding was explained by introducing 3NF's, such as the Tucson-Melbourne (TM) model [6] or the Urbana IX 3NF (UIX) [7], in the nuclear Hamiltonian.

Effective field theoretical methods in a form of chiral perturbation theory ($\chi$PT) provided a solid basis for a construction of nuclear forces. Consistent chiral NN [8-10] and 3N [11-13] forces have been derived within the $\chi$PT framework. Recently improved chiral NN potentials have been constructed by using appropriate regularization in coordinate space [14,15]. This significantly reduced finite-cutoff artefacts of the nonlocal momentum-space regulator used in [9, 10] allowing us to apply improved forces to higher energy Nd scattering.

In the next section we give some examples where elastic scattering data are compared to standard NN potential predictions alone or combined with different 3NF's. These

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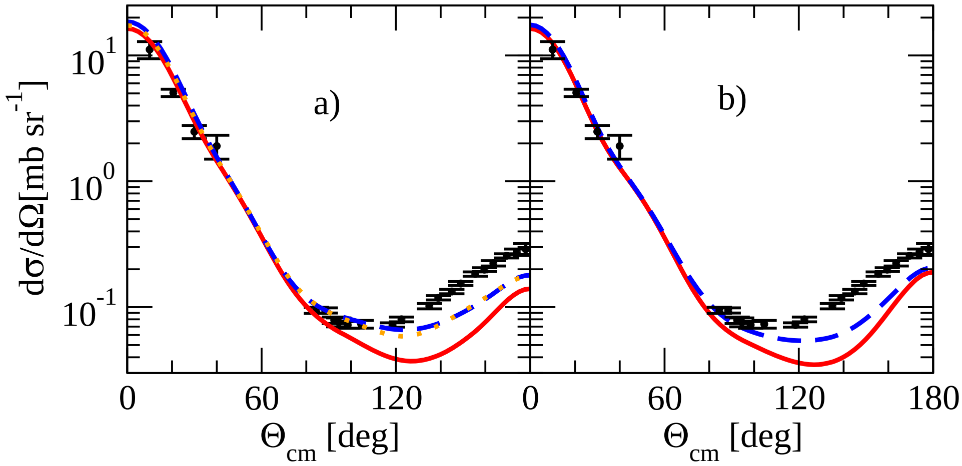


Fig. 1. – The neutron-deuteron (nd) elastic scattering c.m. cross section at neutron energy $E_{lab} = 250$ MeV. The nd experimental data (solid dots) are from [27]. In a) the solid (red) and dashed (blue) lines are predictions of the (semi)phenomenological AV18 potential alone and combined with TM99 3NF [26], respectively. The dotted (orange) line is the prediction of the AV18+Urbana IX 3NF. In b) the solid (red) line is the prediction of the N$^4$LO SCS NN potential with the regulator $R = 0.9$ fm. Combining this NN potential with the N$^2$LO 3NF with $c_D = 6.0$ and $c_E = -1.0943$ provides the result shown by the dashed (blue) line.

examples are chosen to show the importance of the 3NF in the 3N system, whose effects seem to grow with an increasing energy of the 3N system. Therefore we discuss also the role of relativistic effects in that reaction and their significance in the study of 3NF effects. The smallness of relativistic effects, however, indicates that research should focus on 3NF's, especially on those of short-range nature. In the following section we present examples of results for improved chiral NN potentials applied alone or combined with the N$^2$LO 3NF. We summarize in the last section.

## 2. – Results with standard forces

All observables for elastic Nd scattering can be obtained from a state $T|\phi\rangle$, which fulfills the 3N Faddeev equation [16]. Using the realistic NN forces: AV18 [3], CD Bonn [4], Nijm1, Nijm2, and Nijm93 [5] one gets in general predictions for 3N scattering observables which agree well with data at energies below $\approx 30$ MeV. A fairly complete overview of those theoretical predictions in comparison to data is presented in [17, 18]. At higher energies discrepancies develop. The large discrepancy in the minimum of the elastic scattering cross section obtained with NN forces only, seen for energies above $\approx 60$ MeV, is removed for energies below $\approx 140$ MeV when TM or UIX 3NF's, which reproduce the experimental triton binding energy, are included [18, 19]. A similar behavior shows up for the high energy deuteron vector analyzing power $A_y(d)$ [18, 20, 21]. But there are many spin observables for which large 3NF effects are predicted and where the TM and the UIX do not reproduce the data [18]. This is the case e.g. for the nucleon analyzing power $A_y$ [18, 22] and for the deuteron tensor analyzing powers [18]. In none of these cases can the data be reproduced by pure 2N force predictions. Therefore elastic Nd scattering observables can be identified, which are sensitive to the 3NF structure.

## 3. – Relativistic effects in elastic Nd scattering

There are large discrepancies at higher energies between data and theory in elastic Nd scattering which cannot be removed by adding standard 3NF's (see Figure 1a). They require investigations of the magnitude of relativistic effects. We used an instant form relativistic approach which encompasses relativistic kinematics, boost corrections, and Wigner spin rotations [23, 24]. The boost effects turned out to be the most significant ones for the elastic scattering cross section at higher energies. They reduce the transition matrix elements at higher energies and lead, in spite of the increased relativistic phase-space factor as compared to the nonrelativistic one, to rather small effects in the cross section, mostly restricted to the backward angles [23]. Higher energy elastic scattering spin observables are only slightly modified by relativity [23, 24].

## 4. – Results with chiral semilocal coordinate-space regularized forces

The smallness of relativistic effects indicates that very probably the short range contributions to the 3NF are responsible for the higher energy elastic scattering discrepancies. The recently constructed new generation of chiral NN potentials up to N$^4$LO with an appropriate regularization in the coordinate space [14, 15] made it possible to reduce significantly finite-cutoff artefacts bound with the nonlocal momentum-space regulator employed in the chiral NN potentials of Refs. [9, 10]. Applications of these improved semilocal coordinate-space regularized (SCS) NN potentials do not lead to distortions in the cross section minimum of the higher energy elastic Nd scattering that were found in Ref. [25]. Using a new procedure for estimating the theoretical truncation errors introduced in [14,15] provides evidence for missing 3NF effects at higher energy Nd scattering. The expected theoretical uncertainty at N$^3$LO and N$^4$LO is substantially smaller than the observed discrepancies between calculations and data [28,29]. Indeed, combining the SCS N$^4$LO NN potential with the N$^2$LO 3NF regularized in the same way, we found that the predicted 3NF effects in 3N continuum reactions support the results found with standard forces. The nuclear Hamiltonian with a 3NF at N$^2$LO is fixed by specifying the values of constants $c_D$ and $c_E$ which parametrize the strengths of the leading $1\pi$-contact and the three-nucleon-contact terms [11]. To fix them we used first the experimental triton binding energy $E(^3H)$ to determine the dependence of $E(^3H)$ on $c_E$ for a given value of $c_D$. Next, to obtain the specific $c_D$ and $c_E$ values we took the elastic Nd scattering cross section at the incoming nucleon laboratory energy around $\approx 60$ MeV, where clear discrepancies between theory and data develop and pure NN potential predicitions underestimate the data in the region of the cross section minimum up to backward scattering angles [19], Taking a precise pd data set at $E = 70$ MeV from Ref. [30] we performed $\chi^2$ fit of theory to data using ($c_D$,$c_E$) pairs reproducing the experimental value of the triton binding energy and got values of $c_D = 6.09 \pm 0.23$ and $c_E = -1.13 \pm 0.10$. In Figure 1b we exemplify the resulting N$^2$LO 3NF effects in the case of the elastic scattering cross section. It remains to be seen whether consistent 3NF's at N$^3$LO will resolve higher energy discrepancies found in Nd scattering.

## 5. – Summary

Solving 3N scattering exactly in a numerical sense up to energies below the pion production threshold allows one to test the 3N Hamiltonian based on modern NN potentials and 3NF's. At the higher energies for some observables large 3NF effects are predicted when using the (semi)phenomenological 3NF models such as TM and UIX. Some Nd elastic scattering cross sections and polarization data support these predictions. In some other cases, however, defects of the (semi)phenomenological 3NF's are demonstrated. Relativistic effects are found to be small for the elastic scattering cross section and negligible for higher energy spin-observables. The discrepancies at high energies, which remain even when UIX or TM 3NF's are included, point to the importance of short-range contributions to the 3NF. An application of improved chiral NN interactions up to $N^4$LO order of chiral expansion, alone or combined with $N^2$LO 3NF, supports conclusions obtained with the standard forces. Higher order chiral 3NF's comprises a number of shorter-range terms [12, 13]. It can be expected that a use of consistent chiral NN and 3N forces will play an important role in understanding of elastic scattering and breakup data at higher energies.

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