Synthese (2012) 188:117–142
DOI 10.1007/s11229-012-0064-4

A future for the thin red line

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Received: 26 March 2011 / Accepted: 17 January 2012 / Published online: 20 March 2012

Abstract The thin red line (*TRL*) is a theory about the semantics of future-contingents. The central idea is that there is such a thing as the 'actual future', even in the presence of indeterminism. It is inspired by a famous solution to the problem of divine foreknowledge associated with William of Ockham, in which the freedom of agents is argued to be compatible with God's omniscience. In the modern branching time setting, the theory of the *TRL* is widely regarded to suffer from several fundamental problems. In this paper we propose several new *TRL* semantics, each with differing degrees of success. This leads up to our final semantics, which is a cross between the *TRL* and supervaluationism. We discuss the notions of truth, validity and semantic consequence which result from our final semantics, and demonstrate some of its pleasing results. This account, we believe, answers the main objection in the literature, and thus places the *TRL* on the same level as any other competing semantics for future contingents.

Keywords Future contingents · Branching-time · Ockhamism · Thin red line · Supervaluationism

1 Introduction

The thin red line (or *TRL*) is a theory about the semantics of future-contingents. The central idea is that there is such a thing as the 'actual future', even in the presence of (perhaps radical) indeterminism. It is inspired by a well known solution to the

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problem of divine foreknowledge associated with William of Ockham, in which the freedom (and hence moral responsibility) of agents is argued to be compatible with God's omniscience. The original presentation of the modern theory is by Øhrstrøm (1981, 1984), but it is widely held to suffer from fundamental problems (Belnap and Green 1994; Belnap et al. 2001; Placek and Belnap 2010). The *TRL* theory has been developed in several directions (McKim and Davis 1976; Braüner et al. 1998, 2000; Øhrstrøm 2009), but each theory modifies the original in a way that deviates from Ockham's original insight, and its presentation in (Øhrstrøm 1981). We propose several new *TRL* semantics, with differing degrees of success, each of which keeps the *TRL* as unique and fixed. Our final account, the 'supervaluational thin red line' (or STRL), answers the main Belnap-inspired objection that seemed to force theorists away from the *TRL*. We present several pleasing results for the STRL semantics, showing that it generates all the plausible Ockhamist validities and avoids a well-known problem for supervaluationism presented by Willimson and Tweedle. With this begins our project of placing the *TRL* on the same level as other competing semantics for future contingents.

2 Ockhamism

William of Ockham (c. 1288–c. 1348) has been associated with an influential and "widespread point of view" (Zanardo 1996, p. 1) about the semantics of the future-tense in branching time. This might seem surprising, given that Ockham was a 14th Century Franciscan friar and as such predates branching time or formal semantics by about 600 years. We think of him because he wrote about the theological problem of Divine Foreknowledge, which has a strong similarity to the problem of future contingents (perhaps it is precisely the same problem). He claims that God divinely foreknows the future, while at the same time maintaining that we are free agents (with moral responsibility). In very general terms, the *TRL* is a theory which tries to maintain this conclusion (though in a somewhat more secular setting).

2.1 The problem

The problem of Divine Foreknowledge is a traditional theological problem, discussed at length by Ockham in a book entitled *Predestination, God's Foreknowledge, and Future Contingents* (c. 1323, the relevant section is Question 1, section F). What follows is a very simple reconstruction designed to highlight William of Ockham's general solution. The problem can be put very simply: if God already knows what you are going to do tomorrow (i.e. if you are "predestinate"), then you are not free to act otherwise. In order to avoid this conclusion, one seems to have to concede that either agents are not free, or that God is not Omniscient. Ockham refuses to budge, and denies neither of these central tenets of Christianity.[1] There is a delicate interaction

[1] This refusal to budge is indicative of Ockham's obstinate character in general. For example, not long after writing "Predestination", Ockham was accused of heresy, most likely for a commentary on Lombard's "Sentences". The story is that Ockham was summoned to Avignon to face an official papal commission,

involving modality here, and to see it clearly we need to be careful. The problem has to do with two of God's supposed Divine Attributes: Omniscience and Infallibility; the former meaning that God foreknows the future, and the latter meaning that He cannot get it wrong.

We can put matters a little more formally (and a little less fluently) by saying:

Omniscience: If it will be that ϕ, then God foreknows that it will be that $\phi (F\phi \rightarrow K_G F\phi)$.
Infallibility: If it is divinely foreknown that it will be that ϕ, then it is not possible that it will not be that ϕ $(K_G F\phi \rightarrow \neg\Diamond\neg F\phi)$.

This is equivalent to:

Infallibility2: If it is divinely foreknown that it will be that ϕ, then it is necessary that it will be that ϕ $(K_G F\phi \rightarrow \Box F\phi)$.

Omniscience, Infallibility2 and hypothetical syllogism are sufficient to show that the future is necessary:

$F\phi \rightarrow K_G F\phi$ and $K_G F\phi \rightarrow \Box F\phi$. Therefore, $F\phi \rightarrow \Box F\phi$.

The future is necessary and the freedom of will is just an illusion.

2.2 The solution

Ockham diagnoses a modal ambiguity in the argument. The following natural language sentence-type, "If A then necessarily B", can mean either of the following:

1. If A, then necessarily-B.
2. Necessarily, if A, then B.

According to Øhrstrøm (1984), this distinction has been articulated by many authors throughout history. Notably, Anselm makes this distinction, calling (1) 'antecedent necessity' and (2) 'subsequent necessity'.[2] Ockham's message is that God's infallibility should be stated as follows:

It is necessary that if God foreknows that it will be that ϕ, then it will be that $\phi : \Box(K_G F\phi \rightarrow F\phi)$.

It is the implication that is necessary. This formulation expresses that God never gets it wrong (he is infallible), but it doesn't lead to the content of his knowledge becoming necessary as a consequence.

God is necessarily omniscient, so $\Box(F\phi \rightarrow K_G F\phi)$ holds as well. Therefore, every formula of the form $K_G F\phi$ is necessarily equivalent to the formula $F\phi$, so we could reconstruct the argument by replacing the first with the second, giving us the

Footnote 1 continued
which found 51 charges against him. Although Pope John XXII stopped short of formally condemning Ockham, this leniency did not stop the young Englishman from obstinately deciding that it was in fact the Pope who was guilty of heresy. This episode eventually lead to Ockham's excommunication from the Catholic Church.

[2] Anselm uses the same syntax for both and so mistakenly believes there to be two kinds of necessity involved; one of the benefits of logical symbolism is to see this error.

relatively harmless $\Box(F\phi \rightarrow F\phi)$. The last formulation merely expresses the clichéd expression "Que sera sera", only adding a "Necessarily…" in front. But then, surely, it is a modal logic tautology anyway, so that is absolutely fine.

2.3 Conclusion of Ockham

The philosophical message to be exported from Ockham's solution is that God can know the future, without that foreknowledge making the future events necessary. For example, if God knows that you are going to sin tomorrow, then according to Ockham it is still true that you might not sin. The content of God's knowledge that "you will sin tomorrow" is that although you are not under any compulsion to, and are free to refrain from it, you just will. Because of the equivalence of God's foreknowledge and the truth of a prediction, the message is that there can be a truth about the future which does not collapse to necessity or mere possibility; the 'plain will'. It is a delicate point that Ockham is making. The plain future tense is modally thicker than 'possibly might', but modally thinner than 'necessarily will'.

3 Prior meets Ockham

Ockham's solution to the problem of future contingents is not entirely original. As we saw, Anselm had very similar views. But, the reason we think of Ockham, rather than Anselm, is because of Arthur Prior. In his classic book on tense logic, *Past, Present and Future* (1967, Chap. 7) Prior discusses branching and Ockham's views on future contingents at the same time. The particular issue that Prior focuses on is not quite the scope distinction from above, but something different. Prior focuses on the clash between the intuitive thesis that the past is necessary, and sentences which have a "trace of futurity about them."

The idea that the past is necessary stems from the realisation that we cannot alter the past. Nothing we can do now can effect the outcome of World War II. Now that the Allies won, it is necessary that they won. A natural way to bring this idea into tense logic is to insist that past tensed truths are necessary. However, consider the following past tensed statement:

"That was my last cigarette."

If true, it entails that I will not smoke in the future. But if it is past-tensed and true, then it is necessarily true. This in turn entails that I will necessarily not smoke in the future. But surely, even if I do not smoke in the future, it is not a necessary truth now that I will not. Otherwise it would not be so hard to give up. So, past-tensed truths are necessary truths, apart from the ones which have an explicit or implicit trace of the contingent future about them. These must remain contingent.

To get this result, Prior used the branching model (or 'matrix') that Kripke had suggested to him (see Prior 1967, p. 27). Formally, the branching-time (BT) structure is a pair $\mathfrak{F} = \langle M, < \rangle$ where M is a non-empty set, $<$ is a transitive and asymmetric relation defined on M which satisfies the conditions of backward linearity ($\forall m, m_1, m_2((m_1 < m \wedge m_2 < m) \Rightarrow (m_1 \leq m_2 \vee m_2 \leq m_1))$, where $m \leq m'$ means

$m < m' \vee m = m'$) and historical connectedness ($\forall m_1, m_2 \exists m(m \leq m_1 \wedge m \leq m_2)$). Elements of a set M are *possible moments* which can be thought of as possible, instantaneous stages of the world. The symbol '$<$' represents *earlier-possibly later* relation defined on moments, $m_1 < m_2$ means that m_1 is in the past of m_2 and m_2 is in a possible future of m_1. Both M and $<$ are not only temporal but also inherently modal notions which are meant to represent the interaction of possibility with time. We will call the maximal linearly ordered subsets of M '*histories*' as they correspond to entire possible courses of history.

The language we are working with is a propositional language containing countable, infinite set of propositional variables Var, temporal operators F and P, and historical modalities $\Box$ and $\Diamond$. Atomic propositions are thought of as simple, present tensed declarative sentences, rather than 'untensed' or 'tenseless' (see Müller 2011). Additionally, those atomic propositions have no traces of futurity, they are, so to say, wholly about the present. The notion of valuation of propositional variables in BT-structure $\mathfrak{F} = \langle M, < \rangle$ is understood as a function $V : Var \mapsto \wp(M)$. The BT-model based on $\mathfrak{F}$ is a pair $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$.

The particular novelty with this semantics was that Prior insisted that future tensed formulae must be evaluated not just at a moment m, but also at a particular history h passing through that moment (in every m/h pair, $m \in h$). In order to keep the semantics compositional, all formulae are evaluated at such pairs. Here are the semantic clauses for the tempo-modal propositional language:

Definition 1 (Formula ϕ is Ock-true in model $\mathfrak{M}$, at m/h pair)

1. $\mathfrak{M}, m/h \models^{Ock} p$ iff $m \in V(p)$ where $p \in Var$;
2. $\mathfrak{M}, m/h \models^{Ock} \neg\phi$ iff it is not that $\mathfrak{M}, m/h \models^{Ock} \phi$ ($\mathfrak{M}, m/h \not\models^{Ock} \phi$);
3. $\mathfrak{M}, m/h \models^{Ock} \phi \wedge \psi$ iff $\mathfrak{M}, m/h \models^{Ock} \phi$ and $\mathfrak{M}, m/h \models^{Ock} \psi$;
4. $\mathfrak{M}, m/h \models^{Ock} P\phi$ iff $\exists m'(m' < m \wedge \mathfrak{M}, m'/h \models^{Ock} \phi)$;
5. $\mathfrak{M}, m/h \models^{Ock} F\phi$ iff $\exists m'(m < m' \wedge m' \in h \wedge \mathfrak{M}, m'/h \models^{Ock} \phi)$;
6. $\mathfrak{M}, m/h \models^{Ock} \Box\phi$ iff $\forall h'(m \in h' \Rightarrow \mathfrak{M}, m/h' \models^{Ock} \phi)$.

This semantics guarantees that we stop those past tensed formulae that have a trace of futurity about them from being necessary when they shouldn't be. This meant that Prior's reconstruction of Ockham's position was successful.

The purely tense part of the resulting system, given these semantics, turned out to be exactly the logic of linear time, and the purely modal part turned out to be $S5$. The resulting logic is therefore rather conservative. These qualities have made this semantics attractive to many philosophers and philosophical logicians, including Nuel Belnap, Richmond Thomason, Anil Gupta, Tomasz Placek and Thomas Müller.

However, we feel uneasy about the history parameter of evaluation, and do not want it to play a major role in the semantics of future contingents. But before we part company with such esteemed philosophers, we will try to explain our dissatisfaction. We do not present 'knock-down' arguments against the Priorian-Ockhamist semantics, just interpretational difficulties. (This is not dissimilar to the way that Placek and Belnap (2010, p. 22) describe their own anti-*TRL* arguments).

3.1 Problems with the history parameter

The history parameter in Priorian-Ockhamism is a semantic parameter that needs to be specified in order to give a truth value to a prediction of a future contingent. We accept that there are no technical problems with how this parameter works, but we argue that there are difficulties when trying to understand what this semantic parameter means. The value has to be given somehow and a first thought is that it is fixed by reference to some fact, or set of facts. But which facts? The obvious answer is: any set of facts that includes the fact about which future is the actual one. However, proponents of Priorian-Ockhamism strongly deny that there is any such fact; to them there is no such thing as the actual future. So, we can't simply give the history parameter the value of the actual history, as there is none (as Belnap et al. urge). So it has to be given a value, but any value given can only be arbitrary, or "prima facie". But the truth-values of predictions are not arbitrary; they relate to the states of affairs in the future. It might be that, epistemically, because I don't know the future, each is equivalent. But we are not doing epistemic logic here, rather we are doing temporal logic, and what a given agent knows or not should not affect the construction of our system.

Thomason (1970) makes a telling comment about Priorian-Ockhamism:

> …this [Priorian-Ockhamist semantics] is an unstable view, for its import is that statements in the future tense may be neither true nor false. In particular, they will be neither true nor false unless a unique possible future is posited. Since we may often be in situations in which we have made no suppositions concerning which of a variety of possible futures will come about it should also often be the case that certain statements in the future tense are neither true nor false. (Thomason 1970, p. 271).

Thomason's quote makes it seem like the history parameter is fixed by the facts about which future the speaker of the sentence has 'posited'. Burgess says the following:

> The truth-value of a future tense statement depends on which branch we *think of* as representing the course of events which is actually going to turn out to happen. (Burgess 1979, p. 575, emphasis ours).

In the hands of Burgess, the process of 'positing' or 'supposition' seems to be internal, as it depends on which history we "think of" as the right one. On this picture, the history of evaluation parameter might stand for some kind of inner association we make when we say predictions. So for example, I say "It will rain tomorrow" and, as I do so, I think of a future leading from the present in which it does rain tomorrow. This association makes the prediction true because it rains in the future of which I was thinking. This seems to be the doctrine according to Burgess. We call this idea the 'Inner Baptism' approach. We do not believe that any version of the 'Inner Baptism' Priorian-Ockhamism can be correct. Consider the following scenario:

Samantha and Jonny are in a betting shop. Samantha picks a horse called 'Knobbly Knees' which is scheduled to run in the next race, and places a bet. As she makes the bet, she says to Jonny "Knobbly Knees will win," and while she does so she makes the 'internal supposition' to use a history in which he wins as the value of the history parameter. They sit and watch the race, only to see Knobbly Knees come last. Nevertheless, as a good Inner-Baptist, Sam maintains that she spoke the truth. “Who cares what actually happened? My prediction was associated internally with a winning history, so what I said was true.” When she goes to collect her money, the bookie (quite rightly) refuses to pay. This is because bookies do care about what actually happened, and not about what she was thinking of at the time of the bet. It is what ‘actually happens,’ and not any type of inner association, that decides whether the bet would be paid out. Our first complaint then, is that it seems odd that bet payouts do not correspond to the (Inner Baptist’s) truth of predictions. We think that if you make a true prediction, then a bet about the content of the prediction should (perhaps later on) also pay out. This intuitive idea about the relation between true predictions and successful bets seems to be just incorrectly handled by Inner-Baptism. In fact, making true predictions of future contingents is almost as easy as thinking that your prediction is true.

Imagine that Jonny countered Sam’s prediction by saying “Knobbly Knees will not win,” and that he associated his utterance with a future in which the horse loses. Then, he and Sam will both have spoken the truth, even though they sound very much like they have contradicted each other. We find this situation counter-intuitive. Our complaint here is that it seems that only one of Sam or Jonny could have spoken the truth, and the other falsity.

Belnap et al. (2001) present a more sophisticated version of Priorian-Ockhamism. It should be made clear that Belnap et al. do not endorse a version of Inner Baptism. They reject the idea that the history parameter can be fixed with reference to any element of context, such as internal positing etc. It is more like the assignment of values to individual variables; a parameter but not one given by context; it is an auxiliary parameter.

Part of the novelty of this approach lies in their “Semantic Thesis 6–6” (*ibid.*, p. 155), where “The coin will land heads” (without specifying the value of the history parameter) is compared with “x is brindle” (without specifying the semantic value of the variable x). Each are then claimed to have no truth-value, but for the simple reason that they are underspecified. If we fully specify the variables required by the semantics, the truth-value gaps go away (*ibid.*, p. 156). This suggestion is made to increase the similarity between the history and assignment parameters. This comparison is not beyond question, however. Being told that “x is brindle” is clearly a useless thing to be told. After being told it, you do not know which thing is brindle. Being told that “The coin will land heads” just doesn’t seem similarly useless. It seems to be all you needed to know (provided you trust the speaker) in order to confidently make a bet. Therefore, to us, the claim that they should be treated as equally empty seems rather strange.

Belnap et al. are aware of the problem we are alluding to in this section, and suggest a *pragmatic* (as opposed to *semantic*) solution. Their claim is basically that the semantic treatment of the two sentences is the same, but the pragmatic treatment is different. Roughly: “the coin will land heads” is *assertable*, in contrast to “x is brindle” which isn’t (*ibid.*, p. 157), precisely because saying the first, and not the second, brings the speaker into a set of relationships with his audience; he is either ‘impugned’ or ‘vindicated’ as a result of what happens (see Wilson’s contribution to this issue for discussion of this). They say that this pragmatic element is what explains the uselessness of being told that “x is brindle”; it leads to neither vindication nor impugnment. In contrast, by making the prediction, the speaker has entered into a (loose) set of relationships with the audience (hence the need to specify that we ‘trust’ the speaker); they can blame the speaker for getting it wrong (if it turns out to have been wrong), or they can praise him for getting it right (in the other case).

To us, this move to pragmatics seems to be no help. We are concerned with the way that truth-values are given to predictions of future contingents in Priorian-Ockhamism. The basic problem is that utterances occupy single moments but many histories. Since we have to have both to ascribe a truth-value to a prediction (according to Priorian-Ockhamism), there are many non-trivial ways in which we can evaluate a given prediction. It can be true and false, at the same time, that there will be a sea battle tomorrow. Appealing to pragmatics is just to change the subject, in our opinion. It is as if Belnap et al. would have us consider the pragmatics of assertion involved in “α -aserts-‘The coin will land heads’ ” while what we should actually be concerned with is the semantics of “The coin will land heads.”

4 The thin red line

At this point, we follow the lead of Øhrstrøm (1984, p. 217); “It seems clear to me that Ockham was not an Ockhamist (in Prior’s sense of the word). According to Ockham the [history-independent] truth-value of Fp is a meaningful concept. We cannot know the value (unless it is revealed), but God knows it.”

If Øhrstrøm is right, then to be a proper Ockhamist there could be nothing provisional about the history parameter. This means that to construct the True Ockhamism (*TRL*) we should have the notion of the ‘actual course of history’ as a structural feature of the model, rather than as a parameter of evaluation. This seems all the better, given the awkwardness of any attempt to understand the history parameter.

To construct a proper theory of True Ockhamism, therefore, we need to add to the semantical models we are considering. The addition we make is a distinguished history, called the thin red line (or *TRL*).

A *TRL* structure $\mathfrak{T}$ is a pair $\langle \mathfrak{F}, TRL \rangle$, where $\mathfrak{F}$ is a branching time structure and *TRL* is a distinguished history of the model–the history which represents the actual course of events through time. Notice that *TRL* is one particular history of the model, it is not a functional notion usually used in the literature (e.g. McKim and Davis 1976; Barcellan and Zanardo 1999; Braüner et al. 2000; Øhrstrøm 2009) which changes its value depending on a point of the model. In this way we are faithful to Øhrstrøm (1981, 1984); i.e. the original *TRL*.

This modification of the structure results in some changes in syntax and semantics of our language. The symbols $\Box$ and $\Diamond$ are sometimes substituted by $\boxed{F}$ and $\Diamond_{F}$ respectively. We include both $\boxed{F}$ and $\Diamond_{F}$ in the language since under some of the available interpretations these two are not interdefinable. The letter F in $\boxed{F}$ and $\Diamond_{F}$ is meant to indicate that these modalities are inherently ‘future-oriented’ ones. We substitute them with the ordinary $\Box$ and $\Diamond$ when the reference to the future is no longer essential. Each time we provide the appropriate semantic definitions which specify the meaning of the connectives.

We are going to present a series of attempts to grasp the notion of the actual future. All of them utilize the concept of *TRL* structure presented above. We present several options, leaving (in our minds at least) the most promising candidate till last. We decided to keep the ultimately unsuccessful attempts in the article for two reasons: First, they are the initial things you would assume would work, but don't. So, our message is cautionary; we hope our negative results will save others the time of investigating those type of options. Secondly, they demonstrate the difficulties faced by adherents of the *TRL*, and partly excuse the slight (sinful) relaxing of our principles that comes with the final option.

In the next section we outline the basic *TRL* theory and history-independent semantics, in order to introduce the objection we aim to face. We explain what we call the 'conservative response', which is little more than articulating the treatment of non-actual predictions the basic theory gives. As it is clearly inadequate, we then motivate our first option, 'Would'; the second option is called the 'Modal-Would'; the third is the 'Modal-Will'; and lastly we rest on the 'Supervaluational-Will'.

5 Elementary history-independent semantics

The basic *TRL* semantics, found in Øhrstrøm (1981), is based on an idea that we need to intimately bind the interpretation of the F operator with the *TRL*. So, "There will be a sea battle" is true if there is a sea battle in the actual future. Since it is only the $F\phi$ truth clause which is dependent on the choice of the history, if we purge this particular truth clause of the sinful history parameter, then we may attempt to cast aside the history parameter as an element of an evaluation point altogether. (Which formally means to get rid of h on the left side of '$\models$' symbol.) Definition 2 below provides a natural semantics that gets across this idea. A *TRL*-model is a pair $\langle \mathfrak{T}, V \rangle$, where $\mathfrak{T}$ is a *TRL*-structure and $V : Var \mapsto \wp(M)$.

Definition 2 (Formula ϕ is trl-true in *TRL*-model $\mathfrak{M}$, at moment m)

1. $\mathfrak{M}, m \models^{trl} p$ iff $m \in V(p)$ where $p \in Var$;
2. $\mathfrak{M}, m \models^{trl} \neg\phi$ iff it is not the case that $\mathfrak{M}, m \models^{trl} \phi$ ($\mathfrak{M}, m \not\models^{trl} \phi$);
3. $\mathfrak{M}, m \models^{trl} \phi \wedge \psi$ iff $\mathfrak{M}, m \models^{trl} \phi$ and $\mathfrak{M}, m \models^{trl} \psi$;
4. $\mathfrak{M}, m \models^{trl} F\phi$ iff $\exists m'(m' > m$ and $m' \in TRL$ and $\mathfrak{M}, m' \models^{trl} \phi)$;
5. $\mathfrak{M}, m \models^{trl} P\phi$ iff $\exists m'(m' < m$ and $\mathfrak{M}, m' \models^{trl} \phi)$;
6. $\mathfrak{M}, m \models^{trl} \diamond\!\!\!\!F\,\phi$ iff $\exists m'(m' > m$ and $\mathfrak{M}, m' \models^{trl} \phi)$;
7. $\mathfrak{M}, m \models^{trl} \boxed{F}\phi$ iff $\forall h(m \in h \Rightarrow \exists m'(m' \in h \wedge m' > m \wedge \mathfrak{M}, m' \models^{trl} \phi))$.

Notice that these definitions not only bind F with *TRL*, but additionally render $\boxed{F}$ and $\diamond\!\!\!\!F$ essentially tempo-modal operators. To evaluate a formula containing $\boxed{F}$ or $\diamond\!\!\!\!F$ we need to take into account both a temporal factor (a future moment m) and modal factor (the history in which the moment is situated). The intended meaning of $\diamond\!\!\!\!F$ is 'possibly in the future' or simply 'it might be that' and $\boxed{F}$ can be read as 'necessarily in the future' or 'it is inevitable that'. Simply put: modal operators look 'sideways' at other histories but also 'forwards' into the future. The future tense operator looks only forwards, and only into the actual future.

6 Problems for the *TRL*

So far so good, but Belnap et al. (2001) present this theory with various problems; some conceptual, some 'logical'. On the conceptual side, the claim is made that the *TRL* entails determinism (see Thomason 1984; Belnap et al. 2001). We feel that this objection is confused. The whole point of the *TRL* is that the actual future is contingent. It is perfectly consistent in our *TRL* semantics to say "It will (actually) happen, even though it might not," and so we cannot see where the lack of indeterminism could come from. 'Real possibilities' are present in our models. Just because you will actually win the lottery, does not make it necessary that you will. This is Ockham's point. Therefore we believe this objection is not really an objection; the *TRL* is conceptually compatible with indeterminism.

On the more formal side, we face two main problems. The first problem is that the *TRL* provides 'no account' of predictions made off the *TRL*. The second is that the *TRL* has 'problems with actuality.' The first of these problems is largely formal, and it is to this that we sacrifice the four proposals (Sects. 7.1–7.3 below), each of which *can* account for predictions made off the *TRL*. The first three face difficulties, but we believe that the final one gives a rather satisfactory treatment of non-*TRL* predictions. This answers the objection of Belnap et al. that there is no satisfactory treatment of such predictions. We say: Yes, there is. The charge that the *TRL* misconceives actuality will be dealt with in a planned subsequent paper.

Let us first focus in more detail on the first of the aforementioned problems. Belnap et al. say the following about the *TRL* theory:

> The *TRL* theory sounds all right, but it is not. It has the 'logical' defect that it gives no account whatsoever of predictive speech acts occurring at moments of use that lie off the *TRL* and is by so much useless. (Belnap et al. 2001, p. 162).

The theory, as it stands, presents only one course of events as actual, and the semantics of the future tense requires that predictions be checked against later sections of the *TRL*. If the moment of speech is off the *TRL*, then no later moments are in the *TRL*, and so the theory seems to be of no help. Of course, it is not quite correct of Belnap et al. to say that we have "no account whatsoever" of predictions off the *TRL*. Every prediction outside the *TRL* is rendered false by the basic semantics. For any formula ϕ, any *TRL*-model $\mathfrak{M}$ and any moment $m \notin TRL$, $\mathfrak{M}, m \not\models^{trl} F\phi$, $\mathfrak{M}, m \not\models^{trl} F\neg\phi$ and $\mathfrak{M}, m \not\models^{trl} F(\phi \vee \neg\phi)$. This account seems to be a self-evidently Bad Thing to Belnap et al.

Perhaps the result becomes easier to accept if we keep in mind that the whole point of introducing the *TRL* into the model was to represent the notion of the actual future. Therefore, the meaning associated with the F operator, when bound to the *TRL*, should be "it will *actually* be the case that." Under this conservative interpretation of the operator, we should no longer be surprised by the falsity of non-actual predictions. Non-actual moments do not have actual futures, therefore no sentence about the actual future should be true at these moments. We can simply say that the complaint of Belnap et al. asks for what cannot be done.

We believe that Belnap et al. would not be impressed with this reply. They think that even at moments off the *TRL*, we should call *some* predictions true, and not call them all false. The thought might be that even merely possible coins have to land either heads or tails.[3] The *TRL* theory seems to deny this.

There are two general ways in which this intuition can be cashed out. First, for all m off the *TRL* either $F\phi$ is true or $F\neg\phi$ is true; either "there will be a sea battle" is true, or "there will be no sea battle" is true. Second, we can simply require that, for all m off the *TRL*, that $F(\phi \vee \neg\phi)$ is true; "there will be either a sea battle or no sea battle" is true. Call the first the Strong Intuition and the second the Weak Intuition. It is not entirely clear which of these two intuitions is motivating Belnap et al.'s complaint here.

7 Solutions to the first problem

The core of Belnap et al.'s objection is the difficulty of giving an accurate account of predictions made at non-actual moments. In natural language, such predictions are often made by means of counterfactuals. The proper analysis of counterfactuals in general, and in context of branching time semantics in particular, is a rather complex issue (see e.g. Thomason and Gupta 1980; Placek and Müller 2007). Belnap is openly sceptical about formal counterfactual connectives (see his contribution to this issue) and so is plausibly read as not requiring that defenders of the *TRL* provide a full theory of counterfactuals. Nonetheless, Ockhamism (which Belnap et al. endorse) does provide an account of the truth value of predictions made at any point, and one which doesn't make them all false. In order to keep up with Belnap et al. (which would be a self-evidently Good Thing), we intend to do the same; i.e. we will account for predictions situated at moments not in the *TRL* (without rendering them all false), but we will not attempt to provide a semantics for the counterfactual connective. We intend to produce a sequel to this paper in which we will give the proper semantics for the counterfactual connective which includes the *TRL*, and so we ask the reader to suspend his or her worries that rely on counterfactuals.

7.1 Introducing "would"

Our aim in this section of this paper is quite modest. It is to find a way of giving non-actual moments (at least some) true predictions. To begin with, we will keep the future tense as it is, focused solely on the *TRL* and create a new, and somewhat artificial, tense operator 'would'. The idea is that 'would' handles predictions off the *TRL*. So, if England score before half time, they 'will' go on to win the match. In contrast: had England scored before half time, they 'would' have gone on to win. The distinction is pretty clear, but the challenge is to find some plausible semantics for this new operator.

[3] Nuel Belnap agreed with this interpretation in correspondence.

7.1.1 To shift or not to shift

Belnap et al.'s objection involves a telling phrase:

> "Had things gone otherwise".

In Belnap et al.'s own theory, as we have seen, there is a mobile history parameter. The phrase "Had things gone otherwise" means that we simply change the value of the history parameter (and probably also the moment parameter). Braüner et al. (1998) devise a modification of the *TRL* theory, in which there is a *TRL* for every point in the tree. Obviously such a move is not available to us, puritan Ockhamists, but we need to come up with a way of thinking of non-actual predictions, while keeping the *TRL* fixed, not using a mobile history parameter and not invoking multiple *TRL*'s. There don't seem to be many options.

We first thought of using multiple models, each of which has a fixed *TRL*. Each model is thought of as identical apart from the placement of the *TRL*. In short, the idea was to evaluate predictions made at moments off the *TRL* by looking at corresponding moments in other models through which the *TRL* does pass. In this way, it seems, we should be able to avoid the mobile history parameter and get some purchase on non-actual situations (where the *TRL* goes over a different history). This treatment is rather non-standard, as the analysis of object-language sentences involves many semantic models. Nonetheless, it is not a prima facie inconsistent move. However, after incorporating the idea of many *TRL*-models into a formal semantic framework for future contingents, it quickly becomes obvious that this treatment is equivalent to the history-dependent Ockhamist semantics. There would be no 'history' parameter in this semantics, but there would be a ' *TRL*-model' parameter. Just like how the choice of history was arbitrary in the history-dependent semantics, the choice of *TRL*-model is also arbitrary. In terms of the semantic role that they play, the *TRL*-models are just "histories in disguise," and so this move is not going to help. In fact, it is considerably more complicated than the history-dependent semantics, and so this theory may be seen as even worse. There is a temptation to stray from the idea of the *TRL* but we must resist it. Claiming that we should keep the *TRL* fixed while also playing around with multiple *TRL*-models, is rather like professing to a monotheistic faith while also praying to the pagan idols.

We believe that this lesson also concerns the 'Molinist' theories of Øhrstrøm, Braüner and Hasle, in which the semantic model has multiple *TRL*s. We think that the introduction of the alternative models, or multiple *TRL*s, was just an epicycle which did not fix the problem but just pushed it one step further. In our opinion, instead of pushing the difficulty away, one should openly face it at the very beginning.

7.1.2 Modal 'would'

The lesson from the previous section is that we need to face Belnap et al.'s objection in a way that does not, even unconsciously, render the *TRL* parameter mobile. We need to keep it as a fixed, language-independent part of the model. The lesson is: do not introduce to your language any operator which requires you to switch the value of the *TRL* parameter.

However, if we both keep the fixed and unique *TRL* and also do not introduce any analogue of the history parameter at a point of evaluation, then it is very difficult to give an account of the Strong Intuition. This said that at a non-actual moment, either $F\phi$ or $F\neg\phi$ should be true. But it is hard to make one of them true, rather than the other, without privileging a non-*TRL* history somehow.

As strict *TRL* puritans, we must resist the temptation to satisfy Belnap et al.'s strong intuition. Our mantra here is that "there is just no such thing as the actual future of a non-actual moment." Our reason for this view is because of a shared metaphysical conviction we both hold; it is the passage of time that resolves future contingents one way or the other. At the same time, the passage of time, no matter how long-lasting, will never resolve a non-actual future contingent in a similar manner. To the philosophical logician who holds the view, there is therefore a requirement to treat actual future contingents and merely possible future contingents differently. This idea may be considered the 'core' of our view, and so, just to be clear, here is an example:

Imagine I hold in my hand a fair coin. I don't flip the coin but I could have done so. Moreover, I could have said, just before the possible toss, that the coin would show heads. Belnap asks whether this possible statement is true of false. To us, because it is a fair coin and it wasn't flipped, it seems that this assertion cannot be counted as true. Neither would it be true if we substitute 'tails' for 'heads'. Each result is just an unactualised possibility for a fair coin toss that never happened. This is how to think of non-actual predictions of future contingents according to the *TRL*. In contrast imagine that I will flip the coin, and in advance I assert "When I flip the coin, it will show heads." This assertion certainly can be counted as true or false, even in advance of the coin flip (or so we say, and Ockham). The reason for the difference, of course, is the presence of the *TRL*. It is because of this that we can say that the actual future coin flip will 'simply' land heads (although it doesn't have to).

For the reasons above therefore, we think that the rejection of the Strong Intuition is conceptually defensible. Nonetheless, our opinion is that Weak Intuition cannot (and should not) be so easily dismissed. The truth of the Weak Intuition is independent of the belief that one of the possible futures is "quasi-actual". It is grounded in the fact that the 'modal facts' (i.e. which things are possible, inevitable etc.) can be discerned without positing extra *TRL*s.

Imagine that the coin is rigged, so that it has heads on both sides. Again, I don't flip it, but I could have done it and I could have said in advance that the coin would land heads. This possible assertion seems correct to us. This intuition can be accommodated in our semantics because it does not require that the non-actual moment has a *TRL* passing through it. This coin's landing heads is inevitable (i.e. happens in every possible future of the possible flip) precisely because it is rigged. Rigged coins plainly have different modal properties to fair coins, and it is this that allows us to make non-actual predictions about them. Predictions like these are merely possible, but they are not about future contingents and so they can have some kind of answer.

So, we want a definition of 'would' which doesn't require moving the *TRL*. There are two ways to express modal strength of "would" ($W^{\Box}$ and W^{F}), which correspond to whether or not to include the original *TRL* (via the F operator) in the definition:

Definition 3 (‘It would be the case that ϕ’ is true at moment m)

1. $\mathfrak{M}, m \models^{trl} W^{\Box}\phi$ iff $\mathfrak{M}, m \models^{trl} \boxed{F}\phi$;
2. $\mathfrak{M}, m \models^{trl} W^{F}\phi$ iff ($\mathfrak{M}, m \models^{trl} \boxed{F}\phi$ or $\mathfrak{M}, m \models^{trl} F\phi$).

Both versions behave in the same way on merely possible moments; they inherit the meaning of the future oriented necessity operator. However, they differ when evaluated on the thin red line, $W^{\Box}$ still takes the meaning of $\boxed{F}$ while W^{F} follows the actual future. This distinction is subtle, but important.

Let us consider the following example to illustrate the difference. John and Anna played a game of chess (on the *TRL*) and Anna won. However, these two are more or less equally skilful chess players, both could have won. Michel, unaware of the fact that the game was played, formulates the following judgement, which he considers to be counterfactual: “Had John and Anna played chess, Anna would have won.” Is he right?

Let us consider the moment m when John and Anna are beginning the game. At this moment, it is true they both might win, and it is true that Anna will actually win. Then, W^{F}(*Anna is winning*) is true, but $W^{\Box}$(*Anna is winning*) is not. The intuition behind W^{F} is that if an antecedent of the counterfactual is actually true, then ‘would’ should behave as ‘will’. On the other hand, $W^{\Box}$ has a constant, modally strong meaning, even for the actual moments. Since the intuitions about the truth of the sentences as Michel’s above are shaky, we decided to include some investigation of both options.

7.1.3 Problems with modal ‘would(s)’

The construction of the modal would was based on what we think are good intuitions about non-actual predictions; if the non-actual prediction is about something inevitable it should be treated differently from non-actual predictions that are about contingent things. Both of these are different from actual predictions. The purpose of explaining the modal would’s semantic properties therefore was partly to highlight these Good Things and make them clear. However, when you look at the logical consequences of such a semantics, you find that it fails rather spectacularly. We include in the Appendix (1) a list of these shortcomings. The list of difficulties which we point to is by no means exhaustive. One could raise more objections of this sort. We call them “logical” since they are mostly to do with counter-intuitive interaction between the connectives of our language under the proposed semantics.

Take it from us, this theory is killed off completely by our complaints. The interested reader can consult the gruesome details of its death, but the more squeamish (or less interested in going through the list of troublesome examples) can simply pass on to our next idea.

7.2 Modal ‘will’

One way around some of the troubles pointed to in the previous section is to modify the meaning of the future connective F so that it gets the meaning which was previously reserved for W^{F}. We call this the ‘modal-will’. This move allows us to retire the ‘would’ operator completely. The ‘modal-will’ semantics for F is as follows:

Definition 4 (‘It will be the case that ϕ’ is true at moment m)

$$\mathfrak{M}, m \models^{trl} F\phi \text{ iff } ((\exists m'\ m' > m \wedge m' \in TRL \wedge \mathfrak{M}, m' \models^{trl} \phi) \text{ or } \mathfrak{M}, m \models^{trl} \boxed{F}\phi)$$

The idea here is that ‘it will be that ϕ’ is true at m iff either m is in the *TRL*, and ϕ is true later in the *TRL*, or m is not in the *TRL* and ϕ is inevitable at m. The F operator is thus enriched to be sensitive to both modal and *TRL* notions.

The ‘philosophical’ flaw of this solution is that the semantics of F is no longer so tightly entangled with the thin red line, and so we lose some of the purity which was provided by the strict interpretation. This operator can no longer be interpreted simply as ‘actually will’ as its role is no longer to simply refer to the unique and distinguished future which is ahead of us, as it also refers to the settled future of non-actual moments.

However, the gain is quite significant. First of all, we do not need to introduce an independent ‘would’ operator which takes care of predictions made at non-actual moments. The semantics of ‘will’ is sufficiently rich to cover such cases. As a result, we can consent to the thesis that ‘would’ is simply a superficial, grammatical modification of ‘will’ and not an independent operator, which seems intuitively right to us.

By the same token, we do not give up on our claim that actual and non-actual predictions should be treated differently. This semantics respects our idea that to assess the truth value of an actual sentence about the future one just needs to (wait and) see what the actual future is like, while the assessment of the truth value of the non-actual prediction demands something different—namely reasoning about what would be possible and what would be necessary at this non-actual moment. Secondly, acceptance of this new definition of F rescues us from most of the intuitive difficulties we discussed above and which resulted from assenting to the strict, puritan reading of future operator.

The problems (cf. Appendix (1). Replace W_F with F.) we are left with are: 5,6,7,12,13, plus G is not the dual for F and also the quasi-deterministic $F\phi \rightarrow \boxed{F}\phi$ is always true off the *TRL*. If this was the end of the line (and at one point, we thought it was), then there might not be a future for the thin red line. Luckily we devised another tactic.

7.3 Supervaluational thin red line

Our last semantic proposal gives away even more of our philosophical chastity with respect to the *TRL*. But in advance, we get a semantics which we believe to be very accurate in precisely describing the language of an indeterminist who believes in the actual future. Regardless of our sinful digression from the pure theory, our final idea is still very strongly pervaded with the idea of the distinguished, actual course of events, and so is rightly to be called true Ockhamist theory; a genuine *TRL* theory.

We are giving away a part of our general attitude since we appeal to the notion of truth value at the moment/history pairs; the very notion we argued against in Sect. 3.1! However, we do it only provisionally, as a mean of arriving at the final, history independent semantics. So we use it only as a technical tool do define our *TRL* semantics which itself is independent of the history parameter. We are essentially still exploiting the distinction between the different ways of treating actual predictions of future contingents and non-actual ones which we have built up so far in this paper. What we add here, is the supervaluational treatment of truth into the picture. In exchange for the slackening of our principles that this move entails, we can fix the logical faults of the previous approaches. The resulting semantics, in our opinion, accurately mimics the intuitions we have about the interaction of tense, possibility and actuality.

The aforementioned, provisional reference to the history parameter enables us to return to the well known basic tempo-modal language and to appeal in our semantics to the Ockhamist notion of truth (Definition 1). Now, the *TRL*-truth and *TRL*-falsity of a formula are defined as follows:

Definition 5 (*TRL*-truth; *TRL*-falsity)
We say that formula ϕ is *TRL*-true in *TRL*-model $\mathfrak{M}$, at moment m ($\mathfrak{M}, m \models^{TRL} \phi$) iff

$$\forall h (m \in h \Rightarrow \mathfrak{M}, m/h \models^{Ock} \phi) \text{ or } \mathfrak{M}, m/TRL \models^{Ock} \phi$$

We say that formula ϕ is *TRL*-false in *TRL*-model $\mathfrak{M}$, at moment m ($\mathfrak{M}, m \dashv^{TRL} \phi$) iff

$$\forall h (m \in h \Rightarrow \mathfrak{M}, m/h \not\models^{Ock} \phi) \text{ or } \mathfrak{M}, m/TRL \not\models^{Ock} \phi$$

What this says is that at m, it is true that ϕ iff either every m/h pair makes ϕ true, or if the m/TRL makes ϕ true. Formula ϕ is false iff either every m/h pair makes ϕ false, or if it is false on m/TRL. A formula lacks a truth value otherwise. Evidently, a formula ϕ is *TRL*-false iff its negation is *TRL*-true ($\mathfrak{M}, m \dashv^{TRL} \phi$ iff $\mathfrak{M}, m \models^{TRL} \neg\phi$). Notice that the definition borrows from the definition of the weak Modal Would and the Modal Will the idea of making the definition sensitive to concerns that require quantifying over histories and also (if it is present) to the *TRL*. So this last idea is a development of the previous two ideas. Notice that the clause for truth and falsity is disjunctive; either it is super-true (à la Thomason) OR it is true on the *TRL*. The second disjunct is what we add to a standard supervaluational account of truth in branching time (Thomason 1970, 1984).

Clearly our treatment differs from Thomason's in some crucial respects. First of all, the notion of *TRL* plays a role in defining the truth and falsity of the formulae. Hence, it is not the case that all the histories are "prima facie" as Prior put it; one of them is special. This ensures our Ockhamist credentials. Second, it very closely mimics our intuitions about the different status of actual and non-actual predictions that we described above. Each single prediction actually made (i.e. made on the *TRL*) is either true or false at the moment at which it is being made, future contingents included. In contrast, the prediction $F\phi$ can be considered true or false at a non-actual moment m only if the state of the world at the moment m renders either ϕ or $\neg\phi$ inevitable; otherwise, it has no truth value whatsoever. In our opinion it imitates well our ordinary intuitions. Belnap et al.'s complaint was that in the original theory, predictions made at non-actual moments are all false. Now, predictions made about settled truths at non-actual moments are true (as they should be) and predictions about contingents are neither true nor false. This also seems right. This is why there is no answer to which way the possible fair coin would have landed. Importantly, we no longer face the technical difficulties associated with the modal-would or the modal-will. We believe therefore that this semantics is the right way to go when addressing Belnap et al.'s complaints about non-actual predictions.

Note that the intended interpretation of $\mathfrak{M}, m \models^{TRL} \phi$, where m is off the *TRL*, is not "had m been actual, then ϕ would have been true" (which is a subjunctive conditional), but something like "m is the actual world's potential in which ϕ happens." This is not to dogmatically rule out subjunctive conditionals, but just to say that the meta-language interpretation of the evaluation of a formula off the *TRL* should not be subjunctive by default. If one wants an account of subjunctive conditionals, then the task would be to enrich the object language to contain a counterfactual operator, not impose it as a reading of the basic evaluation of non-subjunctive formulae (such as atomic propositions off the *TRL*).

Let us end with a few comments on the formal properties of the semantic definitions we have just proposed.

7.3.1 Validity

Part of the quarrel about the *TRL* consisted in finding various "logical" difficulties for different versions of the *TRL* semantics. These difficulties usually took a form of observations that some intuitively valid formulae are not valid in a given *TRL* semantics (the *TRL*-invalidities discussed in the literature contain for example: $F\phi \vee F\neg\phi$, $\phi \rightarrow HF\phi$, $FF\phi \rightarrow F\phi$, and $F\Diamond\phi \rightarrow \Diamond F\phi$, (cf. e.g. Belnap and Green 1994; Barcellan and Zanardo 1999; Braüner et al. 2000; Belnap et al. 2001; Øhrstrøm 2009). All of these examples are Ockhamist-valid which seems to privilege (at least "logically" or "linguistically") the Ockhamist approach to logic of indeterminate future. The notions of validity we use here are the standard ones:

Definition 6 (*TRL* validity) Formula ϕ is TRL valid in a *TRL*-structure $\mathfrak{T} = \langle M, <, TRL\rangle$ ($\mathfrak{T} \models^{TRL} \phi$) iff for every model $\mathfrak{M} = \langle M, <, TRL, V\rangle$ based on $\mathfrak{T}$ and every moment $m \in M$, $\mathfrak{M}, m \models^{TRL} \phi$.

Definition 7 (Ockhamist validity) Formula ϕ is Ockhamist valid in a BT-structure $\mathfrak{F}$ ($\mathfrak{F} \models^{Ock} \phi$), if it is true in every BT-model $\mathfrak{M}$, at every moment/history pair m/h.

These notions of validity can be naturally generalized to classes of appropriate structures.

Our proposal gets the better of the previously mentioned "logical" problems since:

Fact 1 *For arbitrary BT-structure $\mathfrak{F} = \langle M, <\rangle$ and TRL-structure $\mathfrak{T} = \langle M, <, TRL\rangle$ based on $\mathfrak{F}$:*

$$\mathfrak{F} \models^{Ock} \phi \Rightarrow \mathfrak{T} \models^{TRL} \phi$$

Proof See Appendix (2a). □

Unfortunately, the converse does not hold in general.

Fact 2 *There is TRL-structure $\mathfrak{T} = \langle M, <, TRL\rangle$, BT-structure $\mathfrak{F} = \langle M, <\rangle$, and formula ϕ such that:*

$$\mathfrak{T} \models^{TRL} \phi \text{ and } \mathfrak{F} \not\models^{Ock} \phi$$

Proof See Appendix (2b). □

However, if we generalize the notion of validity to the level of all *TRL* and BT-structures, the two notions are equivalent. In fact, the equivalence holds even at the lower level of generalization:

Fact 3 *Let* $\mathfrak{F} = \langle M, < \rangle$ *be a* BT *structure and* $\mathbb{T}$ *a collection of TRL structures "based on"* $\mathfrak{F}$ *(i.e.* $\mathbb{T} := \{\mathfrak{T} : \mathfrak{T} = \langle M, <, h \rangle$ *for some history* h *in* $\langle M, < \rangle\}$*), then for any formula* ϕ*:*

$$\mathbb{T} \models^{TRL} \phi \Leftrightarrow \mathfrak{F} \models^{Ock} \phi$$

Proof See Appendix (2c). □

7.3.2 Semantic consequence

Another concept we want to discuss is that of semantic consequence. It is particularly important since lots of arguments against supervaluationism available in the literature (e.g. Timothy Williamson 1994; Tweedale 2004) are focused on counter-intuitive results of the acceptance of supervaluational version of semantic consequence.[4]

We believe that the supervaluationist, against Williamson's (1994, p. 152) warning, should accept the so-called "local" notion of semantic consequence. The *TRL* supervaluationst is no exception. In our case, the local semantic consequence is the Ockhamist one (just as the 'local' truth is the Ockhamist).

Definition 8 (Local semantic consequence) Let $\mathfrak{F} = \langle M, < \rangle$ be a BT-structure, and Γ a set of formulae. We say that formula ϕ is a semantic consequence of Γ in a structure $\mathfrak{F}$ ($\Gamma \models^{Ock}_{\mathfrak{F}} \phi$) iff for every BT-model $\mathfrak{M}$ based on $\mathfrak{F}$:

$$\forall m \forall h \quad (\forall \psi \in \Gamma \ \mathfrak{M}, m/h \models^{Ock} \psi \Rightarrow \mathfrak{M}, m/h \models^{Ock} \phi)$$

More generally $\Gamma \models^{Ock} \psi$ iff $\Gamma \models^{Ock}_{\mathfrak{F}} \psi$ for arbitrary $\mathfrak{F}$.

The first advantage of this approach is that it preserves all the classical rules of inference. Moreover, the notions of semantic consequence and implication are very closely connected ($\phi \models^{Ock} \psi$ iff $\models^{Ock} \phi \rightarrow \psi$). Another benefit of this definition, often unnoticed in the discussion of supervaluationism, is that it preserves a natural analogy between the notions of truth and consequence. If 'super-truth' is defined in terms of truth at every precisification, then 'super-consequence' should be derived from consequence at every precisification.

We should notice though, that the local definition of semantic consequence given above is not always accepted in the context of supervaluationism. The particularly common, global alternative is:

Definition 9 (Global supervaluational consequence) Let $\mathfrak{F} = \langle M, < \rangle$ be a BT-structure, and Γ a set of formulae. We say that formula ϕ is a global supervaluational semantic consequence of Γ in a structure $\mathfrak{F}$ iff for every BT-model $\mathfrak{M}$ based on $\mathfrak{F}$:

[4] We would like to thank Prof. Fabrice Correia and an anonymous reviewer for drawing our attention to various aspects of Williamson's and Tweedale's arguments and their possible impact on our endeavour.

$$\forall m(\forall \psi \in \Gamma \forall h\ \mathfrak{M}, m/h \models \psi \Rightarrow \forall h\ \mathfrak{M}, m/h \models \phi)$$

In the context of BT, such definition was endorsed by Thomason (1970). Someone who believes that this is an accurate supervaluational definition of semantic consequence might urge that the *TRL* version should be an analogous, global one.

Definition 10 (Global *TRL* consequence) Let $\mathfrak{T} = \langle M, <, TRL \rangle$ be a *TRL*-structure, and Γ a set of formulae. We say that formula ϕ is a global *TRL*-semantic consequence of Γ in a structure $\mathfrak{T}$ ($\Gamma \models_{\mathfrak{T}}^{G-TRL} \phi$) iff for every *TRL* model $\mathfrak{M}$ based on $\mathfrak{T}$:

$$\forall m(\forall \psi \in \Gamma\ \mathfrak{M}, m \models^{TRL} \psi \Rightarrow \mathfrak{M}, m \models^{TRL} \phi)$$

An interesting fact about the global *TRL*-consequence is that it behaves considerably better than its supervaluational cousin. To see it, let us remember that Timothy Williamson (1994) famously argued against the supervaluational approach, in the context of vagueness. One of his complaints was very general in nature. He showed that the supervaluational notion of semantic consequence leads to unnatural conclusions, even to, "in a sense a violation of classical propositional logic" (1994, 151). The problem stems from the supervaluationist's preference for 'super-truth' (i.e. truth on all admissible precisifications). According to Williamson, the type of semantic consequence interesting for supervaluationist, is the global one, i.e. the preservation of super-truth. He intends to show that there are argument forms which, while being locally valid, are not globally valid. He shows that the inference rules of *contraposition, conditional proof, argument by cases, and reductio ad absurdum* are not valid given the supervaluational notion of semantic consequence. In our case, 'super-truth' is 'history-independent truth', and the claim would be that these inference rules are not history-independently valid. Pleasingly, our *TRL*-supervaluationism does not suffer from these problems. The *TRL* actually comes to the rescue, as we shall go on to demonstrate.

We will explain one of the arguments to give the reader a sense of the problem, and to see how we escape from it. Contraposition says that if $\phi \models \psi$, then $\neg\psi \models \neg\phi$. The global supervaluational version of this argument doesn't always work though. For instance, if we substitute 'p' for 'ϕ' and '$Definitely : p$' for 'ψ', then $p \models Definitely : p$ holds while $\neg Definitely : p \models \neg p$ does not hold (just because it isn't definitely the case, doesn't mean it isn't the case). This constitutes a counter-example to contraposition, so contraposition is not a law of metalogic of standard supervaluationism. These results apply *mutatis mutandis*, to Thomason's (1970) semantics for indeterministic time. In this case, we can (globally) infer $\Box F\phi$ from $F\phi$, but we cannot infer $\neg F\phi$ from $\neg\Box F\phi$. Therefore, Thomason's version of supervaluationism is vulnerable to Williamson's attack.

In the *TRL* version of global consequence however, the problem does not arise. Remember that in our case, 'super-truth' is the history-independent truth proposed in Definition 5. Unlike Thomason, we allow that $\exists\mathfrak{M}\exists m$ ($\mathfrak{M}, m \models^{TRL} \phi$ and $\mathfrak{M}, m \not\models^{TRL} \Box\phi$). This is a consequence of the second disjunct in the clause defining *TRL*-truth; the sea-battle might only happen on the *TRL* and no other branch. In this situation, the prediction is history-independently true, but the sea battle is not inevitable. It means that, in our account, Williamson cannot make the first step of his argument ($\phi \models^{G-TRL} \Box\phi$ does not hold in general). The contingent sea-battle is a counter-example to this schema. The way of avoiding each of Williamson's three other attacks is the same; each time it is the difference that the *TRL* makes which helps out (see Chap. 5 of Williamson's book "Vagueness", especially pages 151–152, for descriptions of the other arguments).

Another problem for the supervaluationist notion of global consequence was raised by Tweedale (2004) who noticed that $\phi, \Diamond\psi \models \Diamond(\phi \wedge \psi)$ is a valid inference rule in standard supervaluationism. It is a bad result since if we substitute Fp for ϕ and $\neg Fp$ for ψ we get that $Fp, \Diamond\neg Fp \models \Diamond(Fp \wedge \neg Fp)$. So, we can infer a logical impossibility from logically possible set of assumptions. Again, the global *TRL* analogue avoids this problem, $\phi, \Diamond\psi \models^{G-TRL} \Diamond(\phi \wedge \psi)$ is false. To see that, just imagine that ϕ is a future contingent true at $m \in TRL$, then we have that $m \models^{TRL} \phi$, $m \models^{TRL} \Diamond\neg\phi$ but $m \not\models^{TRL} \Diamond(\phi \wedge \neg\phi)$ (we even have that $m \models^{TRL} \neg\Diamond(\phi \wedge \neg\phi)$).

The fact that our *TRL* semantics defends itself against these arguments seems to be a benefit of our theory. It is also pleasing to see that it is the presence of the *TRL* that comes to the rescue. However, despite the global *TRL* consequence relation gets around the problems that the ordinary global supervaluationist consequence relation suffers, we are reluctant to lend our name to it and we still prefer the local notion. Our reason stems from the observation that even though the arguments, as stated in the literature, do not harm our proposal, they can be reformulated in an arguably less persuasive, but still quite severe form. Our semantics avoids the standard problems of supervaluationism because it works at some moments in non-supervaluational manner. Nonetheless, we need to remember that at all moments outside the *TRL* the semantics is entirely supervaluational. One might try to exploit its partially supervaluational character and reconstruct the arguments in a moment-oriented fashion:

Definition 11 (Global *TRL* semantic consequence at moment *m*) Let $\mathfrak{T} = \langle M, <, TRL\rangle$ be a *TRL*-structure and let $m \in M$. We say that ϕ is a global semantic consequence of a set of formulae Γ in structure $\mathfrak{T}$, at moment m ($\Gamma \models^{G-TRL}_{\mathfrak{T},m} \phi$) iff for every *TRL*-model $\mathfrak{M} = \langle M, <, TRL, V\rangle$ based on $\mathfrak{T}$ we have that

$$\forall\psi \in \Gamma\ \mathfrak{M}, m \models^{TRL} \psi \Rightarrow \mathfrak{M}, m \models^{TRL} \phi$$

It is not the notion often met in the literature, but one can give it some intuitive reading. It encodes what follows from what at a given point of a structure, independently of how the valuation function works. Since our language is tensed and the structure might differ from point to point, this notion might be helpful. For example, at any maximal moment m of a structure $\mathfrak{T}$, we have that $F(\phi \vee \neg\phi) \models^{G-TRL}_{\mathfrak{T},m} \psi$ for arbitrary ψ even though $F(\phi \vee \neg\phi) \not\models^{G-TRL}_{\mathfrak{F}} \psi$. Importantly, this notion of semantic-consequence-at-a-moment enables us to exploit the supervaluational characteristics of non-actual moments and establish moment-dependent arguments à la Williamson or Tweedale. For example, for some m outside *TRL* of a structure $\mathfrak{T}$: $\phi \models^{G-TRL}_{\mathfrak{T},m} \Box\phi$, but $\neg\Box\phi \not\models^{G-TRL}_{\mathfrak{T},m} \neg\phi$ and $Fp, \Diamond\neg Fp \models^{G-TRL}_{\mathfrak{T},m} \Diamond(Fp \wedge \neg Fp)$. These arguments do not seem to be as strong as their more general versions, but they might still be found worrisome.

To sum up, the notion of local semantic consequence that we accept is free of logical charges against supervaluationism found in the literature. Additionally, the global version of *TRL* semantic consequence is untouched by arguments as stated which is a fair result. It is important to stress that it is the existence of *TRL* that takes the wind out of the critics' sails. Only a quite unusual moment-dependent modification of the notion of semantic consequence raises some worries.

7.3.3 Truth operator(s)

There is always considerable amount of worry while introducing 'It is true that' and 'It is false that' operators. In our case, the problem does not consist in a danger of semantic paradoxes but in uncertainty of the intended interpretation of these operators. Since there are two levels of semantic analysis—bottom level of history-dependent truth and top level of history-independent truth—there are two competing alternatives for how to understand the truth (and falsity) operator.

Let us first discuss the option that attempts to mimic the top level notion of true on the bottom semantic level. The semantic clause is as follows:

Definition 12 ('It is true that ϕ' is true at m/h) Let ϕ be a formula and $\mathfrak{M}$ a *TRL*-model, then:
$\mathfrak{M}, m/h \models^{Ock} Tr\phi$ iff $\mathfrak{M}, m/TRL \models^{Ock} \phi$, if $m \in TRL$, or
$\forall h'(m \in h' \Rightarrow \mathfrak{M}, m/h' \models^{Ock} \phi)$, if $m \notin TRL$.
We define 'It is false that' (Fl) as $Fl\phi := Tr\neg\phi$; and 'It is undetermined that' (Und) as $Und\phi := \neg Tr\phi \wedge \neg Fl\phi$.

There are some seemingly pleasing consequences of such defined notions. For example we have that for every $m \in TRL$ and every formula ϕ, $m \models^{TRL} Tr\phi \vee Fl\phi$, while for every future contingent evaluated at $m \notin TRL$, we have that $m \models^{TRL} Und\phi$. However, the controversial outcomes far outgrow the merits.

The first set of problems has to do with the semantic consequence relation. In the previous section we accepted the 'local' notion of semantic validity (Definition 8), we argued however that our theory can be combined with the global notion of semantic consequence (Definition 10) and, contrary to classical supervaluationism, it does not generate troublesome results. It was only a moment-dependent notion of global semantic consequence (Definition 11) which has shown to cause some problems. However, if we endow our basic-level semantics with above-defined truth operator, we import the problems of the global supervaluational consequence to the global *TRL* consequence. For example $\phi \models^{G-TRL} Tr\phi$ always holds while $\neg Tr\phi \models^{TRL} \neg\phi$ does not; similarly, $\phi, Und\neg\phi \models^{TRL} Und(\phi \wedge \neg\phi)$ is true. As a result, it seems that either the global notion of semantic consequence or the truth operator defined above needs to be abandoned.

Another set of problems is generated by the fact that Tr behaves differently at actual and non-actual moments. The consequences are quite bizarre. For example, for some models and some moments out of the *TRL* the following formulae *are not* true:

- $Tr(\phi \vee \psi) \rightarrow (Tr\phi \vee Tr\psi)$;

- $\phi \leftrightarrow Tr\phi$;
- $Tr\phi \vee Tr\neg\phi$;
- $FTr\phi \rightarrow TrF\phi$;
- $Tr\phi \wedge HTrF\phi$.

All of them behave well at any $m \in TRL$, however even at some $m \in TRL$ we have that $m \models^{TRL} \Diamond(\phi \wedge \neg Tr\phi)$. Part of the oddity of these formulae can be explained away by different status of actual and non-actual moments but probably not all of it. What is even more unsettling is that the *meaning* of the language seems to change from one point to another. In particular, the meaning of the operator "It is true that" is quite different depending on whether it operates in actual or non-actual circumstances. As a result, it seems for example that T-schema ($\phi \leftrightarrow Tr\phi$) or bivalence ($Tr\phi \vee Fl\phi$) are only contingently true; they hold in the actual world only. This is an idea which we feel reluctant to accept. What distinguishes actual from non-actual moments is the manner in which we can asses truth value of formulae and not the meaning of the language. Consequently, we believe that the aforementioned definition of truth should be rejected altogether.

Since we want to keep the meaning of the truth operator fixed, we prefer to define it in a unified manner throughout the whole domain. Consequently, we decided to accept the following truth operator:

Definition 13 ('It is true that ϕ' is true at m/h) Let ϕ be a formula and $\mathfrak{M}$ a *TRL*-model, then:
$\mathfrak{M}, m/h \models^{Ock} Tr\phi$ iff $\mathfrak{M}, m/h \models^{Ock} \phi$.
Operator 'It is false that ϕ' is defined as $Fl\phi := \neg Tr\phi$.

It is a definition proposed already by Thomason (1970) and it is hard to imagine a more straightforward and intuitive one. It simply says that the truth conditions of $Tr\phi$ are exactly those of ϕ. Preceding the formula with the phrase "It is true that" simply does not distort its meaning at all. By the same token, "It is false that" behaves just as negation which distinguishes it from the notion given by Definition 12.

The generalization to the top level does not change the natural features of this definition. As far as semantic consequence is concerned, the incorporation of the truth operator into our semantics is no longer harmful. In particular, none of Williamson's or Tweedale's counterarguments apply to the global notion of *TRL* consequence, so it regains its advantage over traditional supervaluationism in this respect.

By the same token, the intuitive validities are restored. All of the troublesome formulae above, $Tr(\phi \vee \psi) \rightarrow (Tr\phi \vee Tr\psi)$, $\phi \leftrightarrow Tr\phi$, $Tr\phi \vee Tr\neg\phi$, $FTr\phi \rightarrow TrF\phi$, and $Tr\phi \wedge HTrF\phi$ are always true. In particular, the alethic version of the Weak Intuition ($Tr\phi \vee Fl\phi$) is valid, which we find a pleasing result. If the truth operator does not change the truth conditions of the formula itself and falsity operator functions just as the negation, it would be quite bizarre to universally accept $\phi \vee \neg\phi$ and to reject $Tr\phi \vee Fl\phi$. Evidently, for an arbitrary moment—actual or not—we can justly assess that a contingent statement is either true or false. The difference between the actual and non-actual moment consist in the fact that at the former, but not at the latter we can assess which of these two ways it is.

8 Conclusion

Belnap et al. presented the original thin red line theory with various objections. In this article we have focused on one in particular: how to account for predictions made off the *TRL*. Our supervaluational-*TRL* theory gives the following account of predictions made off the *TRL*; if the prediction is about something inevitable then it is true, if it is about something contingent it is neither true nor false. On the *TRL*, of course, predictions of contingents can be simply true. The gappiness that comes with supervaluations is restricted to the non-actual moments—the thought being that there are no actual futures of non-actual moments. This sensitivity to the *TRL* gives our version of supervaluationism advantages that other versions lack. We plan to address the other outstanding problems for the *TRL*, including treatment of the proper two-place counterfactual connective and treatment of an 'actually' operator, in subsequent papers. So far though, we have addressed the problem of non-*TRL* predictions, and presented our answer in formal semantics. We hope, therefore, for this to be the beginning of a new future for the *thin red line*.

Acknowledgments Jacek Wawer gratefully acknowledges the support of the MSHE's research grant no. 668/N-RNP-ESF/2010/0.

Appendix

(1) Problems with modal 'would(s)'

The following is a fragmentary list of "logical" problems with the 'modal would', from Sect. 7.1.2. We list various counter-intuitive consequences of the proposed semantics, which can be naturally divided into the groups depending on some properties of an evaluation point:

(a) Off the TRL, for all m, for all $\mathfrak{M}$*:*

1. $\mathfrak{M}, m \not\models^{trl} F\phi$, even if ϕ is $\psi \vee \neg\psi$, i.e. $\mathfrak{M}, m \not\models^{trl} F(\psi \vee \neg\psi)$;
2. $\mathfrak{M}, m \models^{trl} G\phi$ $(G := \neg F \neg)$;
3. $\mathfrak{M}, m \models^{trl} F\phi \leftrightarrow F\neg\phi$;
4. $\mathfrak{M}, m \models^{trl} F\phi \rightarrow \boxed{F}\neg\phi$.

(b) Off the TRL, for some m, for some $\mathfrak{M}$*:*

5. $\mathfrak{M}, m \not\models^{trl} W^F(\phi \vee \psi) \rightarrow (W^F\phi \vee W^F\psi)$;
6. $\mathfrak{M}, m \models^{trl} \phi \wedge \neg H W^F\phi$;
7. $\mathfrak{M}, m \not\models^{trl} W^F\phi \vee W^F\neg\phi$ (even though $W^F\phi \vee \neg W^F\phi$ is valid. Clearly then, $W^F\neg\phi$ and $\neg W^F\phi$ are not equivalent);

8. $\mathfrak{M}, m \not\models^{trl} \boxed{F}\phi \rightarrow F\phi$ (even though on *TRL* it works fine: $\mathfrak{M}, m \models^{trl} \boxed{F}\phi \rightarrow F\phi \rightarrow \Diamond\!\!\!\!F\,\phi$).

(c) On the TRL, for some m for some $\mathfrak{M}$*':*

9. $\mathfrak{M}, m \not\models^{trl} F\phi \rightarrow W^{\Box}\phi$
10. $\mathfrak{M}, m \models^{trl} F\phi \wedge \boxed{F}\phi \wedge \neg\boxed{F}F\phi$

(d) For some m, for some $\mathfrak{M}$*:*

11. $\mathfrak{M}, m \not\models^{trl} \phi \rightarrow HW^{\Box}\phi$. But the weaker version $\phi \rightarrow H\neg W^{\Box}\neg\phi$ still holds.
12. $\mathfrak{M}, m \not\models^{trl} \boxed{F}\phi \rightarrow \phi$
13. $\mathfrak{M}, m \models^{trl} \boxed{F}\neg\phi \wedge \Diamond\!\!\!\!F\,\phi$

Additionally, $\boxed{F}$ and $\Diamond\!\!\!\!F$ are not duals. The dual of $\boxed{F}$ is a "weak" $\Diamond\!\!\!\!G$ which says that "it is possible that it always is going to be the case that" and the dual of $\Diamond\!\!\!\!F$ is a "strong" $\boxed{G}$ saying that "it is necessarily always going to be the case that."

(2) Ockhamist validity and *TRL* validity

(a) Ockhamist validity implies TRL validity

Proof The proof is a straightforward consequence of the notion of Ockhamist truth (Definition 1), supervaluational *TRL*-truth (Definition 5) and validity (Definitions 6 and 7).

Assume that ϕ is Ockhamist valid in a structure $\mathfrak{F}$. This means that for any moment/history pair m/h and any model $\mathfrak{M}$ based on $\mathfrak{F}$, $\mathfrak{M}, m/h \models^{Ock} \phi$. Now, it is sufficient to analyse the notion of truth we are adopting in Definition 5 to notice that for every *TRL*-model $\mathfrak{N}$ based on $\mathfrak{T}$ and every moment m, $\mathfrak{N}, m \models^{TRL} \phi$. Hence, ϕ is *TRL*-valid. □

(b) TRL validity does not imply Ockhamist validity

The counterexample is not that easy to find. Our definitions guarantee that for any *TRL*-model $\mathfrak{N} = \langle M, <, TRL, V\rangle$ and any $m \notin TRL$, if $\mathfrak{N}, m \models^{TRL} \phi$, then in BT-model $\mathfrak{M} = \langle M, <, V\rangle$ "underlying" $\mathfrak{N}$, for any h such that $m \in h$, $\mathfrak{M}, m/h \models^{Ock} \phi$. Additionally, if $m \in TRL$, then $\mathfrak{N}, m \models^{TRL} \phi$ implies that $\mathfrak{M}, m/TRL \models^{Ock} \phi$. Therefore, a counter-example can be found only at m/h pairs such that $m \in TRL$ and $h \neq TRL$. We found such counterexample. Let us consider a rather unusual *TRL*-structure $\mathfrak{T}$ such that the ordering $<$ is dense on the *TRL* and it is also dense "outside the TRL" (i.e. $\forall m_1, m_2((m_1 \notin TRL \wedge m_2 \notin TRL \wedge m_1 < m_2) \Rightarrow \exists m_3\, m_1 < m_3 < m_2))$. Now, assume that there is at least one discrete jump from the *TRL*; i.e. that the following condition holds $\exists m_1, m_2(m_1 \in TRL \wedge m_2 \notin TRL \wedge m_1 < m_2 \wedge \neg\exists m_3\, m_1 < m_3 < m_2)$. In such a peculiar structure it would be *TRL*-valid that $\mathfrak{T} \models^{TRL} F\phi \rightarrow FF\phi$ even though in the underlying BT-structure $\mathfrak{F}$, $\mathfrak{F} \not\models^{Ock} F\phi \rightarrow FF\phi$ (to see that, we just need to pick a moment m on the *TRL* and a moment $n \in h$ immediately after m

outside the *TRL* and assume that $\mathfrak{M}, n/h \models^{Ock} \phi$ and n is the only such moment in h then $\mathfrak{M}, m/h \not\models^{Ock} F\phi \rightarrow FF\phi$). This *TRL*-structure generates additional problems: even though in our structure $\mathfrak{T} \models^{TRL} F\phi \rightarrow FF\phi$, $\mathfrak{T} \not\models^{TRL} H(F\phi \rightarrow FF\phi)$, so the latter is not a global *TRL* semantic consequence (see Definition 10) of the former.

Interestingly, if $P\phi \rightarrow PP\phi$ is valid in *TRL*-structure $\mathfrak{T} = \langle M, <, TRL \rangle$, then relation $<$ is dense. To see this, assume otherwise, i.e. (a) $\mathfrak{T} \models^{TRL} P\phi \rightarrow PP\phi$ and (b) $<$ is not dense, that is, $\exists m_1 \exists m_2 (m_1 < m_2 \wedge \neg \exists m_3\ m_1 < m_3 < m_2)$. Now, consider a model $\mathfrak{N} = \langle M, <, TRL, V \rangle$ such that $V(q) = \{m_1\}$ for some $q \in Var$. Finally, examine two possible cases:

- $m_2 \in TRL$. Since $\mathfrak{T} \models^{TRL} P\phi \rightarrow PP\phi$, we have that $\mathfrak{N}, m_2 \models^{TRL} Pq \rightarrow PPq$. By Definition 5, this implies that $\mathfrak{N}, m_2/TRL \models^{Ock} Pq \rightarrow PPq$. Since, $V(q) = \{m_1\}$ and $m_1 < m_2$, we have that $\mathfrak{N}, m_2/TRL \models^{Ock} Pq$ which implies (by Definition 1) that $\mathfrak{N}, m_2/TRL \models^{Ock} PPq$. It follows that $\exists m'\ m' < m_2$ and $\mathfrak{N}, m'/TRL \models^{Ock} Pq$. Since $V(q) = \{m_1\}$, we have that $m_1 < m' < m_2$ which contradicts assumption (b).
- $m_2 \notin TRL$. Again, we have that $\mathfrak{N}, m_2 \models^{TRL} Pq \rightarrow PPq$. Consequently, $\mathfrak{N}, m_2/h \models^{Ock} Pq \rightarrow PPq$, for arbitrary h such that $m \in h$. Then, we pick any such h and reason just as in the previous case to derive contradiction with (b).

So, there is an asymmetry between P and F in our semantics; in particular $F\phi \rightarrow FF\phi$ is a global *TRL* semantic consequence of $P\phi \rightarrow PP\phi$ (consult Definition 10), but the converse does not hold[5].

(c) Generalized TRL-validity does imply Ockhamist validity

Proof The right to left implication is again a simple consequence of definitions. The converse is very easy to prove as well. Let us assume, for reductio, that for some formula ϕ, $\mathbb{T} \models^{TRL} \phi$ and $\mathfrak{F} \not\models^{Ock} \phi$. It means that there is a model $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$, a moment m, and a history h such that $\mathfrak{M}, m/h \not\models^{Ock} \phi$. Now, let us consider the *TRL*-structure $\mathfrak{T} \in \mathbb{T}$ such that the history h is the $\mathfrak{T}$'s *TRL* and a model $\mathfrak{N} = \langle \mathfrak{T}, V \rangle$, which valuation function is the same as the one in $\mathfrak{M}$. Since, $\mathfrak{M}, m/h \not\models^{Ock} \phi$ and h is $\mathfrak{N}$'s *TRL*, it follows from our definitions that $\mathfrak{N}, m \not\models^{TRL} \phi$ and as a result $\mathbb{T} \not\models^{TRL} \phi$ which contradicts our assumption. □

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[5] We would like to thank an anonymous referee for drawing our attention to this point.

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