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Miracles: A Logical Perspective*

There is an extensive bibliography devoted to the problem of miracles.¹ It is not our goal to analyze those works, particularly as they offer diverse definitions of miracles depending on the accepted conceptual scheme, such as Aristotelian ontology. Our goal is less ambitious: by adopting two explications of the notion of a miracle, we shall try to answer the question of how the knowledge concerning miraculous events may be embedded into one's belief system. In particular, we shall be interested in uncovering the logical mechanisms at work in this process.

1. Introduction

Simplifying considerably, one can argue that miracles are events which are contrary to the common course of nature (*communum cursus naturae*). Medieval philosophers posited that one should distin-

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¹ Cf. e.g. J.T. Driscoll, *Miracle*, [in:] *The Catholic Encyclopedia*, vol. 10, Robert Appleton Company, New York 1911, <http://www.newadvent.org/cathen/10338a.htm> (30 March 2010).

guish between two orders: so-called *ordo naturalis* or natural order, which is fully penetrable by human reason, and *ordo supranaturalis vel miraculis*, or the supernatural order, which can be known only through revelation. From this perspective, miracles – belonging to the supernatural order – are phenomena which cannot be accounted for with the use of natural reason.²

This understanding of miracles leads us to an important controversy. One should ask whether miracles are contrary to the principles governing the universe, or whether they strike us as contrary to the physical laws, but it is just a result of our ignorance. The answer to this question hangs together with how one understands the status of *ordo naturalis* and *ordo supranaturalis*: are they ontological concepts or only epistemological ones. According to the former interpretation, *ordo naturalis* is a set of laws governing the universe, and as a result, a miracle constitutes a violation of those laws. On the latter reading, the distinction between the two *ordines* coincides with the distinction between what can potentially be known through natural reason (*ordo naturalis*) and what reason cannot grasp (*ordo supranaturalis*). Thus, a miracle would be an event which complies with the laws governing the universe, but such laws which human reason cannot comprehend naturally.

The declarations of the Doctors of the Church pertaining to this problem are ambiguous. Augustine claims that miracles are not contrary to nature, but only contrary to what we know about nature. He says:

So great an author as Varro would certainly not have called this a portent had it not seemed to be contrary to nature. For we say that all portents are contrary to nature; but they are not so. For how is that contrary to nature which happens by the will of God, since the will of so mighty a Creator is certainly the nature of each created thing?

² Cf. B. Brożek, *The Double Truth Controversy*, Copernicus Center Press, Kraków, 2010, Chapter 1.4.

A portent, therefore, happens not contrary to nature, but contrary to what we know as nature.³

On the other hand, Aquinas observes in *Summa Theologica* that miracles lie outside of the natural order or “are done outside the order of nature”.⁴ The declaration of Augustine may be interpreted as a confirmation of the epistemological theory of miracles: a miracle is an event which is contrary to our knowledge of the universe (or, more precisely, contrary to what we can potentially know about it). The stance of Aquinas seems to support the ontological interpretation: a miracle is an occurrence contrary to the laws governing the universe.

It is not our goal to answer the question of which of these interpretations is correct. We assume that both options are interesting from a logical point of view. Before we proceed with further analysis, however, one more distinction is needed. It seems that one can speak of two kinds of miracles. The first category involves the unrepeatable miracles, that is such that constitute unique events, e.g. when Lazarus was raised from the grave. The second category comprises the repeatable miracles, an example being the Eucharistic transubstantiation. Such repeatable miracles can be captured by a general ‘law’ (e.g. each time wine and bread are consecrated during a holy Mass, they become the blood and body of Christ).

The question we venture to answer is: what is the formal structure of theological knowledge if it allows the existence of miracles? In order to carry out our analysis, let us formulate the following two examples:

Example 1: a repeatable miracle. According to Aristotle, there are two kinds of entities: substances, which exist *simpliciter* (*per se*), and accidents, which exist *secundum quid* or *in* substances. It is not possible for an accident to exist *per se*. Thus, it is impossible to explain

³ Augustine, *The City of God*, Book XXI, Chapter 8.

⁴ Thomas Aquinas, *Summa Theologica*, I:110:4.

what happens during transubstantiation. The teaching of the Catholic Church is that after transubstantiation the accidents of bread and wine exist – miraculously – without an underlying substance.

Let us formalize this example in the first order logic.⁵ Let ACC stand for ‘is an accident’, DEP – ‘is dependent in its existence’, SUB – ‘is a substance’, IND – ‘is independent in its existence’, and HOS – ‘is an accident of bread after transubstantiation.’ Thus, we get:

- (1) $\forall x (ACC(x) \rightarrow DEP(x))$ *premise (natural reason)*
- (2) $\forall x (SUB(x) \rightarrow IND(x))$ *premise (natural reason)*
- (3) $\forall x (IND(x) \equiv \neg DEP(x))$ *definition*
- (4) $\forall x (HOS(x) \rightarrow IND(x))$ *premise (faith)*
- (5) $\forall x (HOS(x) \rightarrow ACC(x))$ *premise (natural reason)*

Example 2: an unrepeatable miracle. It is impossible to raise a dead man from his grave. However, Lazarus was raised from his grave in a miraculous way.

Let us assume that MOR stands for ‘is dead’, and RES – ‘to be resurrected’, while l is an individual constant (a proper name) for Lazarus. We get:

- (i) $\forall x (MOR(x) \rightarrow \neg RES(x))$ *premise (natural reason)*
- (ii) MOR(l)
- (iii) RES(l)

2. Epistemological understanding of miracles

Let us assume, first of all, that miracles should be understood epistemologically and look from this perspective at the repeatable

⁵ We formalize this example in the first order logic, which may seem incorrect since we use individual variables to refer to both accidents and substances. It would be more natural to use the second order logic. At the price of gross simplification, we have decided to utilize the first order logic, which is sufficient for our goals.

miracles as depicted in Example 1. Let h stand for the accident of some particular bread after transubstantiation:

(6) $\text{HOS}(h)$ *premise*

Based on the premises (1)-(6), we may conclude that:

(7) $\text{IND}(h)$ *from (4) and (6)*

(8) $\text{ACC}(h)$ *from (5) and (6)*

(9) $\text{DEP}(h)$ *from (1) and (8)*

(10) $\neg\text{DEP}(h)$ *from (4) and (7)*

Thus, we obtain a contradiction. A way to avoid it is to take into account a piece of information which comes from revelation, namely that the accidents of bread after transubstantiation exist independently in a miraculous way. This knowledge must be incorporated into the general law based on natural reason, according to which accidents have no independent existence. Thus, we must modify the premise (1) in the following way:

(1)* $\forall x ((\text{ACC}(x) \wedge \neg\text{HOS}(x)) \rightarrow \text{DEP}(x))$

Under those new circumstances, it is impossible to derive (9) as it is not true that $\neg\text{HOS}h$. Let us note that such a reformulation of the premise (1) complies with the epistemological understanding of miracles. A miracle – in our case, the independent existence of the accidents of bread after transubstantiation – is not something contrary to the laws of nature, but rather incompatible with what we know about those laws. To put it differently: the revealed truth pertaining to the independent existence of the accidents of bread after transubstantiation constitutes a piece of information about the laws governing the world, and so it must be incorporated into the general law expressed in the premise (1).

It may be added that in a similar way – through the incorporation of exceptions – the knowledge based on the natural reason alone is expanded. Let us consider a general law expressed in the following way:

$$(P) \forall x (R(x) \rightarrow S(x))$$

Let us assume further that we have observed an exception to this rule such that for any x , which is R but also T , x is not S . In such a case, one needs to revise the law (P):

$$(P^*) \forall x ((R(x) \wedge \neg T(x)) \rightarrow S(x))$$

This type of revision is an example of the concretization of a law of physics, i.e. dispensing with some idealizing assumption of that law.

The situation is different in the case of unrepeatable miracles. In our example:

- (i) $\forall x (MOR(x) \rightarrow \neg RES(x))$ *premise (natural reason)*
- (ii) $MOR(l)$
- (iii) $RES(l)$

The contradiction is easy to spot since the premises (i) and (ii), through the elimination of the universal quantifier and *modus ponens*, yield:

$$(iv) \neg RES(l)$$

An attempt to take advantage of the strategy of incorporating exceptions into the general law results, in this case, in the following modification of the premise (i):

$$(i)^* \forall x ((MOR(x) \wedge \neg x=l) \rightarrow \neg RES(x))$$

This modification is unfortunate as it requires for each and every occurrence of an unrepeatable miracle which contradicts the law ex-

pressed in (i) the introduction of a clause ‘ $\wedge \neg x=n$ ’, where n is an individual constant, i.e. a proper name of an object which does not submit to the law (i). This manoeuvre leads to fatal consequences: our knowledge is no longer universal, but becomes ‘casuistic’. Such laws as the one expressed in (i)* are not algorithmically compressible.⁶ Moreover, the exceptions in the form ‘ $\wedge \neg x=n$ ’ are purely redundant in any system of knowledge: they are always formulated *ex post factum* and serve no role in the prediction of future events.

The situation changes little when one – instead of modifying (i) by introducing clauses in the form of ‘ $\wedge \neg x=N$ ’ – opts for the following solution:

$$(i)^{**} \forall x ((MOR(x) \wedge \neg MIR(x)) \rightarrow \neg RES(x))$$

where MIR stands for ‘is subject to a miraculous event’. In order to make inferences on the basis of (i)** one would need to confirm first that the given object x is not subject to a miraculous event, while this may be stated only *ex post*.

The above-presented analysis shows that – on the epistemological interpretation of miracles – only repeatable miracles may be reasonably incorporated into one’s web of beliefs. Unrepeatable miracles, on the other hand, are anomalies, which – introduced into our knowledge – constitute redundant information. It must be noted, however, that since the mechanism of incorporating ‘repeatable miracles’ into our knowledge is – from the logical point of view – the same as the mechanism of concretizing the laws of physics, the distinction between ‘natural’ and ‘supernatural’ exceptions is always based on an extra-logical criterion. Indeed, as our example clearly shows, repeatable miracles are always non-empirical (they cannot be observed). If they were observable, they would be indistinguishable from other – natural – phenomena, which are in conflict with some general law.

⁶ Cf. M. Heller, *Czy świat jest matematyczny?*, [in:] *Filozofia i Wszechświat*, Universitas, Kraków 2006, p. 51.

3. Ontological understanding of miracles

In the case of the ontological interpretation of miracles, the solution outlined above, i.e. that exceptions constituted by miraculous events should be incorporated into the formulation of general law, is unacceptable, since our goal is for our knowledge to mimic the structure of the universe (it should describe the laws governing the reality). Thus, in our Example 1, the premise (1) should be preserved in the form:

$$(1) \forall x (ACC(x) \rightarrow DEP(x))$$

The problem here is that – together with the premises (2)-(6) – the premise (1) yields a contradiction. The only way out of this trouble is to abandon classical logic and admit that the existence of ontologically understood miracles forces one to express the laws depicting the structure of the universe in the so-called defeasible logic.⁷

Such defeasible logic (let us call it DL) operates at two levels. On the first level, from the given set of premises one constructs *arguments*; on the second level, the arguments are *compared* in order to decide which of them *prevails*. The conclusion of the prevailing argument is also the conclusion of the entire set of premises.

The language of DL is the language of the first order logic extended by the addition of a new connective, the so-called defeasible implication \Rightarrow . For the defeasible implication, there is a special inference rule of *defeasible modus ponens*:

$$\begin{array}{l} A \Rightarrow B \\ A \\ \hline B \end{array}$$

⁷ Cf. H. Prakken, *Logical Tools for Modelling Legal Argument*, Kluwer, Dordrecht 1997; B. Brożek, *Defeasibility of Legal Reasoning*, Zakamycze, Kraków 2004.

The difference between the material and defeasible implications is visible only at the second level of DL.

The language of DL serves to construct arguments. Let us have a look at the following example:

- (a) $A \Rightarrow B$
- (b) $C \Rightarrow \neg B$
- (c) A

This set of premises may serve us to construct the following argument:

- (ARG1)
- (a) $A \Rightarrow B$
 - (c) A
-
- (d) B

Let us extend our set of premises by adding the following sentence:

- (e) C

Now, we can construct another argument:

- (ARG2)
- (b) $C \Rightarrow \neg B$
 - (e) C
-
- (f) $\neg B$

Given two such arguments, we can move onto the second level of DL, where we decide which of the sentences B or $\neg B$ – should be the conclusion of our extended set of premises.

On the second level of DL, two concepts play a key role: *attack* and *defeat*. Simplifying considerably, one can say that an argument

ARG_A attacks an argument ARG_B , when the conclusions of both arguments are contradictory. This is the case in our example, where B and $\neg B$ are contradictory, and thus $(ARG1)$ attacks $(ARG2)$. When two arguments attack each other, it must be decided which of them prevails, or defeats, the other. There are many possible ways of comparing arguments. The easiest and most flexible is to assume that there is an externally (extra-logically) given ordering of defeasible implications. One checks what the defeasible implications that served to construct the two competing arguments are and declares that the one which prevails is that whose defeasible implication is higher in the ordering. In our example, the first argument is constructed with the use of the implication $A \Rightarrow B$, and the second: $C \Rightarrow \neg B$. Let us assume that the second implication is higher in the ordering. If so, it is the second argument that prevails in the comparison with the first one, and it is its conclusion, $\neg B$, which becomes the conclusion of our extended set of premises.⁸ Let us assume further that the arguments which are constructed based on no defeasible implications always prevail in comparison with arguments constructed with the use of some defeasible implications.

Let us now apply this simple idea to our Example 1, replacing all material implications which appear in the premises expressing the laws of natural reason with their defeasible counterparts:

- (1) $ACC(x) \Rightarrow DEP(x)$
- (2) $SUB(x) \Rightarrow IND(x)$
- (3) $\forall x (IND(x) \equiv \neg DEP(x))$
- (4) $\forall x (HOS(x) \rightarrow IND(x))$
- (5) $HOS(x) \Rightarrow ACC(x)$
- (6) $HOSH$

Those premises may serve to construct the following two arguments:

⁸ Thus, DL is a non-monotonic logic.

(ARG3)

(6) $HOS(h)$ (4) $\forall x (HOS(x) \rightarrow IND(x))$ (7) $IND(h)$ *from (4) and (6)*(3) $\forall x (IND(x) \equiv \neg DEP(x))$ (10) $\neg DEP(h)$ *from (3) and (7)*

(ARG4)

(6) $HOS(h)$ (5) $HOS(x) \Rightarrow ACC(x)$ (8) $ACC(h)$ *from (5) and (6)*(1) $ACC(x) \Rightarrow DEP(x)$ (9) $DEP(h)$ *from (1) and (8)*

The arguments attack each other, since their conclusions are contradictory. In this particular case, it is (ARG3) that prevails as it takes advantage of no defeasible implication, when (ARG4) uses two such implications. The logical conclusion of our entire set of premises is then:

(10) $\neg DEP(h)$

Example 2 may be formalized in a similar way:

(i) $\forall x (MOR(x) \Rightarrow \neg RES(x))$ *premise (natural reason)*(ii) $MOR(l)$ (iii) $RES(l)$

On the basis of these premises, one can construct two arguments:

(ARG5)

(iii) $RES(l)$

(ARG6)

(i) $\text{MOR}(x) \Rightarrow \neg \text{RES}(x)$

(ii) $\text{MOR}(I)$

(iv) $\neg \text{RES}(I)$

Again, we have a conflict of arguments. In this case, it is argument (ARG5) that prevails since it takes advantage of no defeasible implication. Thus, the logical conclusion of the entire set of premises is:

(iii) $\text{RES}(I)$

This formalization is satisfactory for two reasons. Firstly, it enables one to preserve the form of the laws of nature: one does not need to introduce any exceptions into the formulation of a law of nature, which is in compliance with the ontological understanding of miracles. Secondly, within the defeasible logic one can identify a logical (formal) difference between our knowledge of miracles and our scientific knowledge. In the former case, a miraculous event ‘blocks’ the application of a general law of natural reason; in the latter – when a phenomenon is observed which is incompatible with some general law – one can revise the law by introducing an exception or through some more complex modifications.

An important drawback of the present solution is that the defeasible logic described above does not have a traditionally understood semantics. Instead, it is based on the so-called *argument-based semantics*, in which the concepts of *attack* and *defeat* play the crucial role. However, there is no (logical) interpretation in the traditional sense here, and defeasible implications are ascribed no truth values.

Such an approach is uncontroversial in the context of the medieval debates pertaining to miracles (or to natural and supernatural orders). Medieval philosophers deemed the laws discovered by the natural reason *probabiles*. This concept of *probabilitas* should not be understood, however, in the contemporary sense of ‘probability’,

but rather as expressing that something ‘can be proved’, ‘is justified’ or ‘is with arguments in its favour’.⁹ With such an understanding of the epistemic status of the laws discovered by the natural reason, the defeasible logic equipped with the argument-based semantics is a perfect tool for rendering the medieval intuitions.

From today’s perspective, however, a different solution is needed. One option is to take advantage of the intentional semantics for defeasible logic.¹⁰ An inspiration for this type of formal structure is the semantics for counterfactual conditionals. Such a counterfactual conditional as $P \rightarrow Q$ is true in an actual world if and only if in a subset of possible worlds Z , which are ‘the most similar to the actual world’, Q is true in all those worlds belonging to Z in which P is true. Defeasible implications may be treated similarly. A defeasible implication, say $P \Rightarrow S$, is true if in a subset of possible worlds N , which are ‘the most normal worlds’, S is true in each world belonging to N in which P is true. Thus, one of our defeasible implications, (i) $MOR(x) \Rightarrow \neg RES(x)$, is true if for every x , $\neg RES(x)$ is true in every most normal world in which $MOR(x)$ is true.

Let us observe that the actual world does not have to be one of ‘the most normal worlds’. In order to make inferences on the basis of a defeasible implication such as (i) $MOR(x) \Rightarrow \neg RES(x)$, we *assume* that it is a most normal world. However, if a miraculous event occurs in the actual world, our assumption is undermined and we are no longer entitled to reason with the use of (i). In such a semantics, considerable controversy is connected to the way of determining the set N (of the most normal worlds). This, however, is an extra-logical problem. To put it simply: N is a set of those worlds in which the laws of the natural order are always fulfilled. The most normal worlds are the worlds in which there are no miracles.

⁹ Cf. B. Brożek, *The Double Truth Controversy*, *op. cit.*, Chapter I.

¹⁰ Cf. H. Prakken, G. Vreeswijk, *Logical Systems for Defeasible Argumentation*, [in:] D. Gabbay (ed.), *Handbook of Philosophical Logic*, 2nd edition, vol. IV, Kluwer, Dordrecht 2002, p. 219-318.

It turns out that the ontologically understood miracles are easier to model logically than miracles understood epistemologically. The use of defeasible logic makes it possible to account for both repeatable and unrepeatable miracles. This is achieved at the price of abandoning the classical logic, which is an interesting consequence of accepting the ontological interpretation of miraculous events.

4. Summary

The problem which we have analysed above constitutes an aspect of a broader controversy, namely the question pertaining to the possibility of God's intervention in the course of events. Contemporary theology – and especially its strands inspired by the development of the natural sciences – makes a number of attempts designed to show that God's intervention in the world is possible without violating the laws of physics. Some see a place for this in the probabilistic character of the laws of quantum mechanics. Others regard such proposals as a subtler version of the 'God of the gaps' argument and opt for an alternative solution. Józef Życiński observes:

Instead of God hidden in Heisenberg's uncertainty, or expressed in the so-called physical chaos, we propose a model, in which the role of God immanent in cosmic history is contained in laws of nature as well as in what we metaphorically call the 'boundary conditions.' The expression denotes theologically conceived boundary conditions in which non-physical (i.e. biological, psychic, spiritual) factors are also taken into consideration in a system considered 'from God's point of view' (again metaphor).¹¹

It must be stressed that both formal explications of miracles we have presented are incompatible with the understanding of God's imma-

¹¹ J. Życiński, *The Laws of Nature and the Immanence of God in the Evolving Universe*, "Studies in Science and Theology" 1997, vol. 5, p. 15.

nence in the world as outlined by Życiński. On both interpretations of miracles – epistemological and ontological – miracles remain phenomena which cannot be explained by science. This is incompatible with *methodological reductionism*, which constitutes the backbone of modern science: there are no such observable facts which *a priori* may be deemed inexplicable by science. Such aprioricity is directly related to miracles understood ontologically, where the natural order is ontologically different from the supernatural order. There is no such difference within the epistemological interpretation of miracles. Thus, one may postulate a reformulation of the epistemological concept such that it would comply with the reductionist strategy of science. However, this would lead to the rejection of such revisions of the laws of nature which introduce redundant information, as e.g. replacing $(i)^* \forall x(\text{MOR}(x) \rightarrow \neg\text{RES}(x))$ with $(i)^* \forall x((\text{MOR}(x) \wedge \neg x=l) \rightarrow \neg\text{RES}(x))$. On this new account, miracles would be ‘naturalized’, and the goal of science would remain to formulate general laws which explain – through ‘natural causes’ – why Lazarus was raised from the grave or why in Cana the water was turned into wine. The question is, however, whether we are still speaking of miracles in this case.