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Fixing the parameters of Lattice HQET including $1/m_B$ terms

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The study of the CKM matrix elements with increasing precision requires a reliable evaluation of hadronic matrix elements of axial and vector currents which can be done with Lattice QCD. The tiniest entry, $|V_{ub}|$, can be estimated independently from $B \to \tau\nu$ and $B \to \pi l \nu$ decays. The ALPHA collaboration has undertaken the effort to evaluate non-perturbatively the decay constant f_B and the $f^+(q)$ form factor for q^2 close to $q^2_{\max}$ entering these determinations. Since for the **b** quark $m_b \gg a^{-1}$ for available lattice sizes, an effective description of the **b** quark is necessary. HQET provides an example of the latter. As any effective theory, HQET is predictive only when a set of parameters have been determined through a process called matching. The non-perturbative matching procedure applied by the ALPHA collaboration consists of 19 matching conditions needed to fix all the relevant parameters at order $1/m_B$ of the HQET action and the axial and vector currents. We present a study of one-loop corrections to two representative matching conditions. Our results enable us to quantify the quality of the observables used in the matching procedure.

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Heavy Quark Effective Theory [1] in its basic formulation provides an effective description of QCD with $N_f - 1$ light quarks and a single heavy quark whose mass is much larger than the QCD energy scale $\Lambda_{\rm QCD}$. The heavy quark is treated non-relativistically and processes are described in its reference frame. In order to avoid the ambiguities of the perturbative expansion of HQET in the strong coupling g [2] one may employ non-perturbative techniques such as lattice QCD and consequently lattice HQET [3]. As any effective theory HQET containes several low energy constants which need to be determined in order to match it to QCD and grant it a predictive power. This step called 'matching' should also be performed non-perturbatively [4]. The ALPHA collaboration has set up a non-perturbative matching strategy to determine the needed HQET parameters at order $1/m_b$ [5, 6, 7]. It relies on a set of carefully chosen observables which are precisely computable in lattice QCD as well as in lattice HQET. We describe the results of a one-loop computation which tests the quality of some of these observables. In order to estimate the $1/m_b^2$ contributions we define a quantity R which measures the ratio of the one-loop corrections to their tree-level value of $1/m_b$ terms. The paper is organized as follows: in section 1 we introduce the lattice HQET Lagrangian and the currents as well as higher dimensional operators needed to account for $1/m_b$ corrections, then in section 2 we briefly describe the framework in which the matching observables are constructed and finally in section 3 we discuss the one-loop results and conclude.

1. HQET at next-to-leading order in m_b

The formulation of lattice HQET was thoroughly discussed in [3] and therefore we only quote the relevant formulae. The Lagrangian is a sum of the static part and two $1/m_b$ corrections

$$\mathscr{L}_{\rm HQET} = \mathscr{L}_{\rm stat} - \Big(\omega_{\rm kin}\mathscr{L}_{\rm kin} + \omega_{\rm spin}\mathscr{L}_{\rm spin}\Big) + \mathscr{O}(1/m_b^2), \tag{1.1}$$

It is part of the definition of HQET that the kinetic and chromomagnetic operators enter only as insertions in the static vacuum expectation values, namely for some operator $\mathscr{O}$ we have

$$\langle \mathscr{O} \rangle_{\rm HQET} = \langle \mathscr{O} \rangle_{\rm stat} + \omega_{\rm kin}\sum_x \langle \mathscr{O}\mathscr{L}_{\rm kin}(x)\rangle_{\rm stat} + \omega_{\rm spin}\sum_x \langle \mathscr{O}\mathscr{L}_{\rm spin}(x)\rangle_{\rm stat} \tag{1.2}$$

The HQET operators themselves are also expanded in $1/m_b$. For the axial current we have

$$\begin{aligned}
(A_0)_R &= Z^{\rm HQET}_{A_0}\{\bar\psi_l\gamma_0\gamma_5\psi_h + c_{A_{0,1}}\bar\psi_l\frac{1}{2}\gamma_5\gamma_i(\nabla^S_i - \overleftarrow{\nabla}^S_i)\psi_h + c_{A_{0,2}}\bar\psi_l\frac{1}{2}\gamma_5\gamma_i(\nabla^S_i + \overleftarrow{\nabla}^S_i)\psi_h\},\\
(A_k)_R &= Z^{\rm HQET}_{A_k}\{\bar\psi_l\gamma_k\gamma_5\psi_h + c_{A_{k,1}}\bar\psi_l\frac{1}{2}(\nabla^S_i - \overleftarrow{\nabla}^S_i)\gamma_i\gamma_5\gamma_k\psi_h + c_{A_{k,2}}\bar\psi_l\frac{1}{2}(\nabla^S_k - \overleftarrow{\nabla}^S_k)\gamma_5\psi_h\\
&\quad + c_{A_{k,3}}\bar\psi_l\frac{1}{2}(\nabla^S_i + \overleftarrow{\nabla}^S_i)\gamma_i\gamma_5\gamma_k\psi_h + c_{A_{k,4}}\bar\psi_l\frac{1}{2}(\nabla^S_k + \overleftarrow{\nabla}^S_k)\gamma_5\psi_h\},
\end{aligned}$$

and similarly for the vector current (we use the notation from Ref.[3]). ψ_l denotes a relativistic, massless fermion, whereas ψ_h is a nonrelativistic heavy fermion. In order to define HQET and the currents at the next-to-leading order one has to fix 3 parameters in $\mathscr{L}_{\rm HQET}$ and 2×3 parameters in $A_0(x)$ and $V_0(x)$ and 2×5 in $A_k(x)$ and $V_k(x)$ giving in total 19 parameters. They are usualy denoted collectively by ω_i, with $i = 1,\ldots,19$. In this work we concentrate on the parameters $\omega_{\rm kin}$ and $c_{A_{0,1}}$ and the corresponding matching conditions.

2. How to determine the HQET parameters?

The HQET parameters are determined by considering observables ϕ_i which can be reliably calculated in lattice QCD and in lattice HQET. The matching condition reads

$$\phi_{i,\mathrm{QCD}}(L,z,a=0) \stackrel{!}{=} \phi_{i,\mathrm{HQET}}(L,z,a,\{\omega(z,a)\}) = \phi_{i,\mathrm{stat}}(L,a) + \phi_{ij,1/m}(L,a)\,\omega_j(z,a), \tag{2.1}$$

where L is the size of the finite volume in which the observables ϕ_i are defined, a is the lattice spacing and z is a dimensionless parameter used to fix the heavy quark mass m given by $z = \bar{m}(L)L$, where $\bar{m}(L)$ is the mass defined in the lattice minimal subtraction scheme [8]. In Ref.[9] it was proposed to use observables defined in the Schrödinger functional framework which differs from the usual one by the boundary conditions that are imposed on the fields at time 0 and T (for a more detailed discussion of the Schrödinger functional framework see [8]). Apart from the usual fields in the bulk one has boundary fields which can be used to construct correlation functions. The observables analyzed in this work are constructed from boundary-to-boundary or boundary-to-bulk correlation functions, e.g.

$$F_1(\theta) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}_l'(\mathbf{u})\gamma_5\zeta_h'(\mathbf{v})\bar{\zeta}_h(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\rangle, \tag{2.2}$$

$$K_1(\theta) = -\frac{a^{12}}{6L^6}\sum_i\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}_l'(\mathbf{u})\gamma_i\zeta_h'(\mathbf{v})\bar{\zeta}_h(\mathbf{y})\gamma_i\zeta_l(\mathbf{z})\rangle, \tag{2.3}$$

$$f_{A_0}(\theta,x_0) = -\frac{a^6}{2}\sum_{\mathbf{u},\mathbf{v}} \langle \bar{\zeta}_h(\mathbf{u})\gamma_5\zeta_l(\mathbf{v})\left(A_0\right)_I(x_0)\rangle \tag{2.4}$$

where ζ and $\bar{\zeta}$ denote fermionic fields living on the boundary. The θ angles are additional kinematic parameters corresponding to the momenta of quark fields in the bulk. The observables are defined in such a way as to cancel all renormalization factors and the angles can be tuned such as to minimize cut-off effects [9]

$$\phi_2(\theta_1,\theta_2) = \frac{1}{4}\log\frac{F_1(\theta_1)}{F_1(\theta_2)} + \frac{3}{4}\log\frac{K_1(\theta_1)}{K_1(\theta_2)}, \quad \phi_4(\theta_1,\theta_2) = \log\frac{f_{A_0}(\theta_1,x_0=T/2)}{f_{A_0}(\theta_2,x_0=T/2)}. \tag{2.5}$$

In Ref.[9] the proposed set of matching conditions was solved at tree-level yielding the classical HQET parameters and it was checked that the $1/m_b^2$ corrections are small. The purpose of the present study is to confirm these conclusions by a one-loop computation similar to the one performed in [10].

The one-loop contributions to the observables Eq.(2.5) were calculated with `pastor`, an automatic tool for generation and calculation of lattice Feynman diagrams [11]. It is a flexible package, which takes as input the discretized action, the definition of the correlation function, and parameters such as L/a and the dimensionless heavy quark mass z. Then, `pastor` automatically generates the Feynman rules corresponding to the specified action, all Feynman diagrams corresponding to the requested correlation function and a numerical contribution of each diagram. The calculations were performed for the Wilson plaquette gauge action and $\mathcal{O}(a)$-improved Wilson fermions with two light quarks and one massive. One-loop contributions were evaluated for $\phi_{i,\mathrm{QCD}}$ and $\phi_{i,\mathrm{stat}}$.

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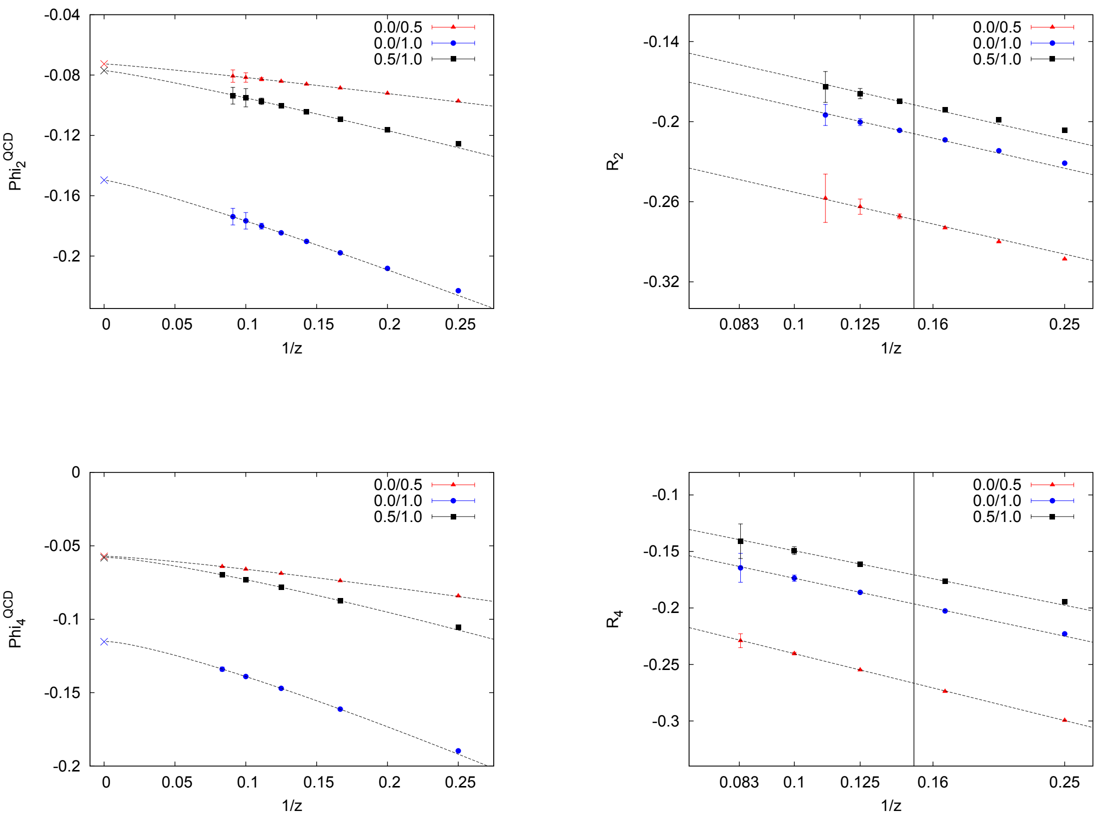

Figure 1: Results for ϕ_2 (up) and ϕ_4 (down). Figures on the left present the z dependence of the one-loop contributions to QCD observables together with their static limit. For the fits we used the functional ansatz with two free parameters f_i and g_i: $\phi^{(1)}_{i,\mathrm{QCD}}(z) = \phi^{(1)}_{i,\mathrm{stat}} + f_i/z + g_i \log(z)/z$. Figures on the right show the corresponding R ratios. Fits were performed using data on the left of the vertical solid line.

3. How to estimate the quality of the observables?

To analyze the $1/z^2$ corrections to an observable ϕ at one-loop order in the coupling constant we expand the matching condition Eq.(2.1) in g^2 and get (dropping terms of order g^4 and $1/z^2$)

$$\phi^{(0)}_{\mathrm{QCD}}(z) + g^2\phi^{(1)}_{\mathrm{QCD}}(z) = \phi^{(0)}_{\mathrm{stat}} + g^2\phi^{(1)}_{\mathrm{stat}} + z^{-1}\sum_t \left(\hat{\omega}^{(0)}_t \hat{\phi}^{(0)}_t + g^2\hat{\omega}^{(1)}_t(z)\hat{\phi}^{(0)}_t + g^2\hat{\omega}^{(0)}_t\hat{\phi}^{(1)}_t\right) \quad (3.1)$$

where the sum over t refers to different subleading contributions. The parameters $\hat{\omega}_t$ differ from the HQET parameters in Eq.(2.1) by an explicit factor $1/\bar{m}(L)$ which was factored out, whereas $\hat{\phi}_t$ have an explicit factor L. In this notation the kinetic contribution is $\hat{\omega}^{(0)}_{\mathrm{kin}} = \frac{1}{2}$, the spin contribution vanishes at tree level ($\phi^{(0)}_{\mathrm{spin}} = 0$), and the remaining contributions correspond to corrections to the current operators proportional to the coefficients c_X. To quantify the $1/z^2$ corrections we define a ration R by extracting the one-loop contribution from Eq.(3.1) by dividing by $(\phi^{(0)}_{\rm QCD}(z) - \phi^{(0)}_{\rm stat})g^2$

$$R(\theta_1,\theta_2) = \frac{\phi^{(1)}_{\rm QCD}(z) - \phi^{(1)}_{\rm stat}}{\phi^{(0)}_{\rm QCD}(z) - \phi^{(0)}_{\rm stat}} = \frac{\sum_t \hat{\omega}^{(0)}_t \hat{\phi}^{(1)}_t(\theta_1,\theta_2)}{\sum_t \hat{\omega}^{(0)}_t \hat{\phi}^{(0)}_t(\theta_1,\theta_2)} + \frac{\sum_t \hat{\omega}^{(1)}_t(z) \hat{\phi}^{(0)}_t(\theta_1,\theta_2)}{\sum_t \hat{\omega}^{(0)}_t \hat{\phi}^{(0)}_t(\theta_1,\theta_2)} \tag{3.2}$$

$$= \alpha(\theta_1,\theta_2) + \beta(\theta_1,\theta_2) + \gamma(\theta_1,\theta_2)\log(z) \tag{3.3}$$

Since the left hand side of Eq.(3.3) has a well defined continuum limit, the right hand side can be also considered in the continuum (the right hand side of Eq.(3.3) must be considered as a entity since the particular terms in the sum may be divergent as $a \to 0$). Note that the explicit $1/z$-dependence cancels. The only z dependence remains in $\hat{\omega}^{(1)}_t(z)$ and can be parametrized as $\hat{\omega}^{(1)}_t(z) = \beta_t + \gamma_t \log(z)$. Hence, when R is plotted on a linear-log plot, the ratio R measures simultaneously:

- $1/z^2$ *corrections*: deviations from a linear behaviour signal $1/z^2$ contributions,
- *slope*: the coefficient of the subleading logarithm.

The slope gives information about a specific linear combination of the anomalous dimensions of the $1/m_b$ operators, which at one-loop order are universal (independent of the scheme) and in some cases can be predicted analytically. We will now present the ratios R for two representative matching conditions.

The simplest matching condition is the one for $\omega_{\rm kin}$ [9]. The z dependence of the one-loop contribution to the observable ϕ_2 is shown on figure 1(a). The ratio R_2 is particularly simple since there is only one subleading contribution, namely $\hat{\omega}_{\rm kin}$,

$$R_2(\theta_1,\theta_2) \equiv \frac{\phi^{(1)}_{2,\rm QCD}(z) - \phi^{(1)}_{2,\rm stat}}{\phi^{(0)}_{2,\rm QCD}(z) - \phi^{(0)}_{2,\rm stat}} = \frac{\hat{\phi}^{(1)}_{2,\rm kin}(\theta_1,\theta_2)}{\hat{\phi}^{(0)}_{2,\rm kin}(\theta_1,\theta_2)} + \frac{\hat{\omega}^{(1)}_{\rm kin}(z)}{\hat{\omega}^{(0)}_{\rm kin}} = \alpha(\theta_1,\theta_2) + \beta + \gamma \log(z) \tag{3.4}$$

Using the fact that in the continuum $\bar{m}(L)\omega^{(0)}_{\rm kin} = \frac{1}{2}$ we have $\hat{\omega}^{(1)}_{\rm kin}(z)/\hat{\omega}^{(0)}_{\rm kin} = 2\bar{m}(L)\omega^{(1)}_{\rm kin}(z)$. From continuum HQET we know (see for example [2]) that reparametrization invariance fixes the renormalization factor for the kinetic operator to its classical value to all orders of perturbation theory. This is true if the quark mass used to define $\omega_{\rm kin}$ is given as the pole mass. In our computation we use the $\bar{m}(L)$ mass, therefore a conversion factor needs to be included. The one-loop conversion between the pole mass and the $\overline{\rm MS}$ scheme can be taken for example from [12] where its 3-loop version was derived, whereas the relation between the $\overline{\rm MS}$ mass and the $\rm MS_{lat}$ was given in [13]. Hence, we obtain the one-loop correction to $\omega_{\rm kin}$ as

$$\bar{m}(L)\omega^{(1)}_{\rm kin}(z) = -\frac{1}{6\pi^2} - \frac{1}{2}0.122282\, C_F + \frac{1}{4\pi^2}\log z, \tag{3.5}$$

from which the parameters in Eq.(3.4) can be obtained, namely, $\beta = -\frac{1}{3\pi^2} - 0.122282\, C_F$ and $\gamma = \frac{1}{2\pi^2}$. The data shown on figure 1(b) exhibits a slope compatible with the predicted one. We can also conclude that $1/z^2$ corrections are equally small for all sets of θ angles, hence the best setting can be choosen by Monte Carlo precision and tree-level considerations.

Figure 1(d) shows the R ratio for the observable ϕ_4, where several terms contribute to the sums on the right hand side of Eq.(3.3) and the θ-dependence of the coefficient of the logarithm doesn't cancel any more. Hence, a seperate fit was performed for each set of θ angles. Again, one concludes that all data lie on straight lines and therefore the $1/z^2$ correction are indeed small.

4. Conclusions

Lattice HQET is a prototype of an effective theory where one can perform a non-perturbative matching. We have studied quantitatively the contamination of the matching conditions by $1/m_b^2$ contributions and confirmed the tree-level conclusion that such corrections are negligible. The ratios R introduced in Eq.3.3 proved to be useful as they provide a handle for the various parts of the $1/m_b$ contributions. Complete results for the remaining matching conditions will be presented elsewhere [14].

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