

## KrkNLO PARTON SHOWER MATCHING\*

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The consistent combination of Next-to-Leading-Order (NLO) perturbative QCD with the logarithmic resummation of parton shower algorithms (‘NLO matching’) is a workhorse of precision QCD in the LHC era. Two methods for achieving this have been widely adopted: MC@NLO and POWHEG. The differences between them are formally Next-to-Next-to-Leading-Order (NNLO) and therefore irrelevant for NLO accuracy, but are nevertheless numerically significant for certain processes and observables. We summarise a third method, KrkNLO, and present preliminary phenomenological results from its implementation in Herwig 7.

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## 1. Introduction

The central objective of ‘NLO matching’ is to produce predictions for exclusive high-multiplicity observables (*e.g.* events with a definite number of observed jets) from an inclusive, low-multiplicity fixed-order NLO calculation, retaining the perturbative accuracy of the latter (up to power corrections) but augmenting it with the logarithmic resummation of a parton shower to provide the flexibility of exclusive predictions. This has been solved in general by the MC@NLO [1] and POWHEG [2–4] methods.

The KrkNLO method was introduced in [5–7] and the results of a preliminary implementation presented for the Drell–Yan and gluon-fusion Higgs production processes in [6, 7]. Here, we extend the implementation to the diphoton process as a stepping stone to a general automated implementation for all colour-singlet final states.

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## 2. The KrkNLO method

The KrkNLO method was introduced in [5–7] and solves the NLO matching problem in a fundamentally different way to its counterparts, exploiting the general freedom to choose a factorisation scheme for parton distribution functions (PDFs) for QCD calculations at NLO and higher. Differences between alternative matching methods, whilst formally higher-order, may be probed to clarify the ‘matching uncertainty’ implied by the choice of any single method.

The KrkNLO method may be summarised succinctly as:

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for all Born events do shower
  if emission generated, from kernel ( $\alpha$ ) then
     $w \leftarrow w \times \frac{R(\Phi_{m+1})}{P_m^{(\alpha)}(\Phi_{m+1})}$ 
  else
     $w \leftarrow w \times \left[ 1 + \frac{\alpha_s(\mu_R)}{2\pi} \left( \frac{V(\Phi_m; \mu_R)}{B(\Phi_m)} + \frac{I(\Phi_m; \tilde{\mu}_R)}{B(\Phi_m)} + \Delta_0^{\text{FS}} \right) \right]$ 
  end if
end for

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The parton shower is then allowed to run to completion. Further details are available in [8].

Here,  $\Phi_m$  denotes the Born phase space and  $\Phi_{m+1}$  the real-emission phase space.  $B$ ,  $R$ , and  $V$  denote the relevant Born, real, and virtual matrix elements, respectively;  $P_m^{(\alpha)}$  the emission kernel used in the shower algorithm to generate the chosen branching of generalised type ( $\alpha$ );  $\mu_R$  the renormalisation scale, and  $I$  the contribution from the shower Sudakov, integrated over the radiative phase space, containing no residual collinear dependence.

This achieves NLO accuracy only when combined with a modification of the PDF factorisation scheme from the usual  $\overline{\text{MS}}$  scheme into the so-called ‘Krk’ scheme [9]. This scheme has so far been formulated only for colour-singlet final states. The transformation between the schemes is defined by the requirement that it compensate for the additional collinear terms generated at  $\mathcal{O}(\alpha_s)$  by the parton shower, which cannot be removed by a simple multiplicative reweighting. This uniquely defines the collinear  $x$ -dependence of the scheme transformation, but does not fix so-called ‘virtual’ terms proportional to  $\delta(1-x)$  which must be fixed by a chosen convention, such as a sum rule [9]. Here, we denote this choice by  $\Delta_0^{\text{FS}}$ . Formally, the change in scheme is  $\mathcal{O}(\alpha_s)$ , so the Krk scheme can also be used for LO calculations.

In order for the full real-emission phase space  $\Phi_{m+1}$  to be populated by the reweighted first shower emission, the shower must be constructed to have full phase-space coverage (*i.e.*, no ‘dead zones’). This can be relaxed using the MC@NLO method to fill in the remaining phase space [10].

Numerically, the KrkNLO procedure shares with POWHEG the advantage of generating no negative-weighted events. These arise in the MC@NLO method due to its use of subtracted, and potentially over-subtracted, events in the hard process. Unlike POWHEG, in KrkNLO, this is achieved without requiring the modification of the shower kernel (and by consequence, the Sudakov factor) for the first parton-shower emission.

### 3. KrkNLO in Herwig 7

Herwig 7 [11–13] is a multi-purpose Monte Carlo Event Generator which provides NLO QCD predictions, matched automatically and consistently to either the dipole [14, 15] or the angular-ordered shower [16], using either the MC@NLO or POWHEG methods, within the Matchbox module [15].

The KrkNLO method is implemented within Herwig 7 separately from the Matchbox methods, as a reweight within the DipoleShower class, currently for a subset of the possible processes. The diphoton process has been implemented manually as an intermediate step to full automation. It has been validated numerically by numerically calculating, and unweighting by, the Sudakov factor generated by the veto algorithm [17–19] and comparing the resulting distributions numerically to the fixed-order real-emission matrix element.

Separately, the virtual matrix elements have been tested, both at the phase-space point level and at the level of distributions, against MadGraph [20] and OpenLoops [21]; the Krk factorisation scheme PDF convolution has been tested numerically against the automated implementation of the Catani–Seymour  $\mathbf{P}$  and  $\mathbf{K}$  operators within Matchbox.

### 4. Diphoton phenomenology

We present results for the KrkNLO method compared against fixed-order NLO and the MC@NLO method, both after the shower has been allowed to generate at most one emission and after the shower has been allowed to run to completion.

We use fiducial cuts corresponding to an idealised ATLAS 13 TeV set-up [22], using smooth-cone (‘Frixione’) isolation [23] with the ‘tight’ isolation parameters of the 2013 Les Houches Accords [24] in place of the experimental isolation and fiducial cuts:

$$p_T^{\gamma 1} > 40 \text{ GeV}, \quad p_T^{\gamma 2} > 30 \text{ GeV}, \quad (1a)$$

$$\Delta R_{\gamma\gamma} > 0.4, \quad |y^\gamma| \in [0, 2.5), \quad (1b)$$

$$E_T^{\text{iso,part}} < 0.1 p_T^\gamma \quad \text{within cone } \Delta R \leq 0.4. \quad (1c)$$

Jets are clustered using the anti- $k_T$  algorithm [25] with a radius of 0.4 and, for jet distributions, are identified as a jet if  $p_T^j > 1$  GeV and  $|\eta_j| < 5$ . The cuts are applied to the final state produced by the generator using Rivet [26].

The MC@NLO and fixed-order comparisons are generated using Matchbox within Herwig 7, with tree matrix-elements computed by MadGraph [20] and loop matrix-elements by OpenLoops [21]. The renormalisation and factorisation scales are set as  $\mu_R = \mu_F = M_{\gamma\gamma}$ . Where the Herwig dipole shower is used, the shower cut-off is set at  $p_T^{\text{cut}} = 1$  GeV and the starting scale is either  $M_{\gamma\gamma}$  (the default) or unrestricted (*i.e.*, a ‘power’ shower [27, 28]). As explained in Section 2, for KrkNLO, the shower starting scale must be unrestricted to ensure full coverage of the real-emission phase space.

We use CT18NLO PDFs [29], either in the  $\overline{\text{MS}}$  scheme or transformed into the Krk scheme for the KrkNLO calculation, and accordingly adopt  $\alpha_s(M_Z) = 0.118$ .

In Fig. 1, it can be seen that the magnitude of the Sudakov suppression in the low- $p_T^j$  region is the dominant source of the differences between the MC@NLO and KrkNLO predictions, just as it is the dominant source of the differences between MC@NLO and fixed-order NLO. The methods converge at high- $p_T^j$ . Within KrkNLO, this arises because the real-emission matrix element is always accompanied by the shower Sudakov factor  $\Delta|_{p_{T,1}}^{Q(\Phi_m)}$ , since the real-emission phase-space point  $\Phi_{m+1}$  is generated by the shower algorithm starting from a Born event; within MC@NLO, the real-emission matrix element is generated independently and so has no accompanying Sudakov factor. This discrepancy is distributed unevenly over phase space and for some distributions leads to significant differences between the methods.

The  $d\sigma/dp_T^\gamma$  distributions can be seen to agree well between fixed-order NLO and MC@NLO, with KrkNLO 10–30% lower; for the  $d\sigma/dp_T^{\gamma 1}$  distribution, the predictions all converge above 100 GeV, whereas for  $d\sigma/dp_T^{\gamma 2}$ , the Sudakov suppression remains relevant at higher values of  $p_T^{\gamma 2}$ . The spike in the fixed-order NLO prediction around  $p_T^{\gamma 2} = 40$  GeV is an unphysical artefact of perturbation theory due to soft gluon emission allowed as a consequence of the asymmetric phase-space cuts on the photons [30, 31].

In Figs. 2 and 3, it can be seen that some of these discrepancies become less significant as the shower evolution proceeds. For the  $d\sigma/dp_T^\gamma$  distributions (*cf.* Fig. 3), there is good agreement between the KrkNLO and the ‘default’ MC@NLO prediction in which the starting scale of the shower is taken to be  $M_{\gamma\gamma}$ , whereas the ‘power’ shower with no upper bound on the radiative phase space gives substantially larger predictions for high- $p_T^{\gamma 1}$ .

At large  $p_T^j$  (*cf.* Fig. 2), the KrkNLO method gives smaller predictions than the default MC@NLO by approximately 40%; this becomes of a comparable magnitude to the deviation between ‘default’ and ‘power’ shower

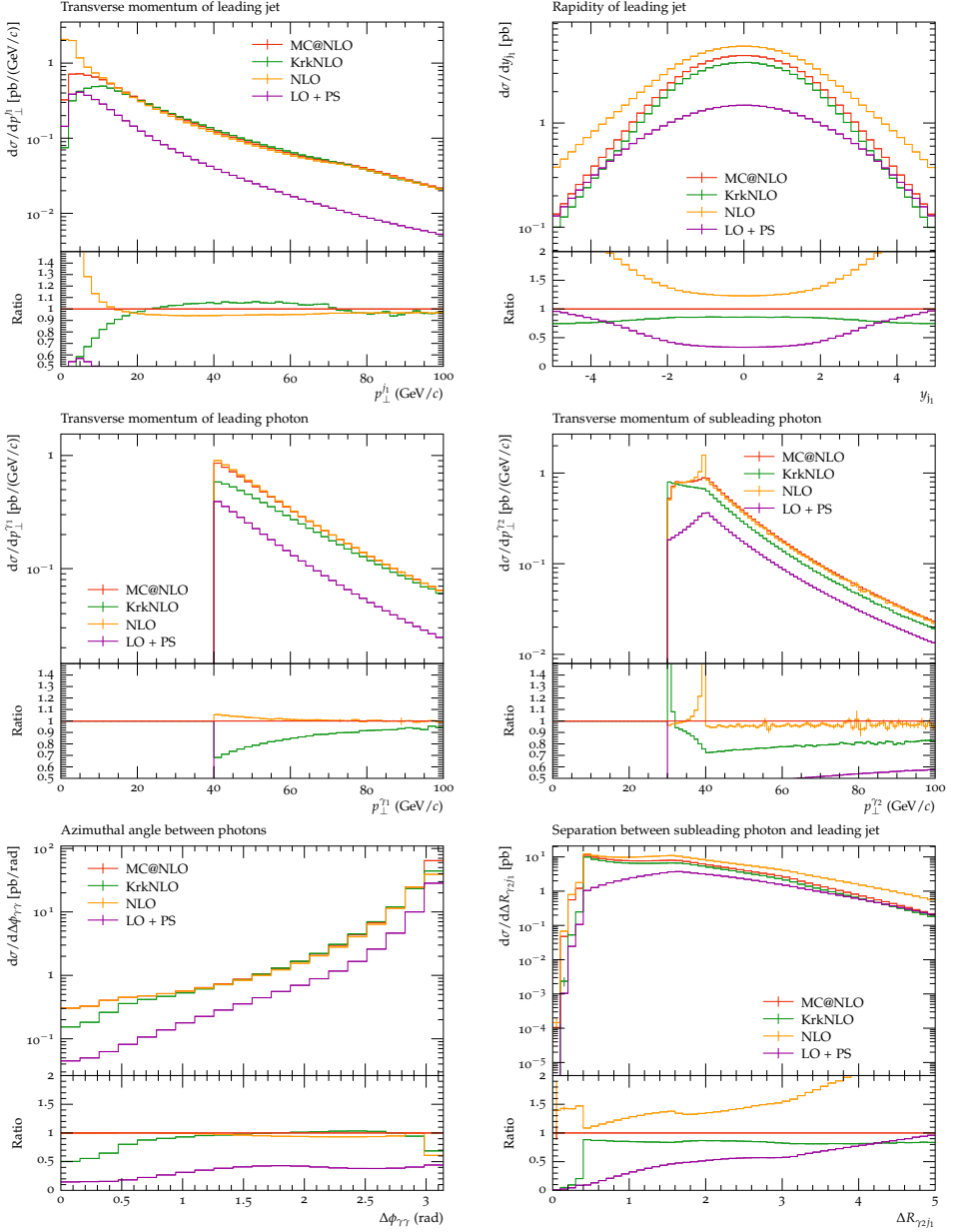


Fig. 1. ‘Parton level’ (first-emission) comparison of KrkNLO with MC@NLO, NLO fixed-order, and the corresponding first-emission distributions generated by the parton shower with a leading-order calculation. The shower in each case is a ‘power shower’, *i.e.* with no phase-space restrictions.

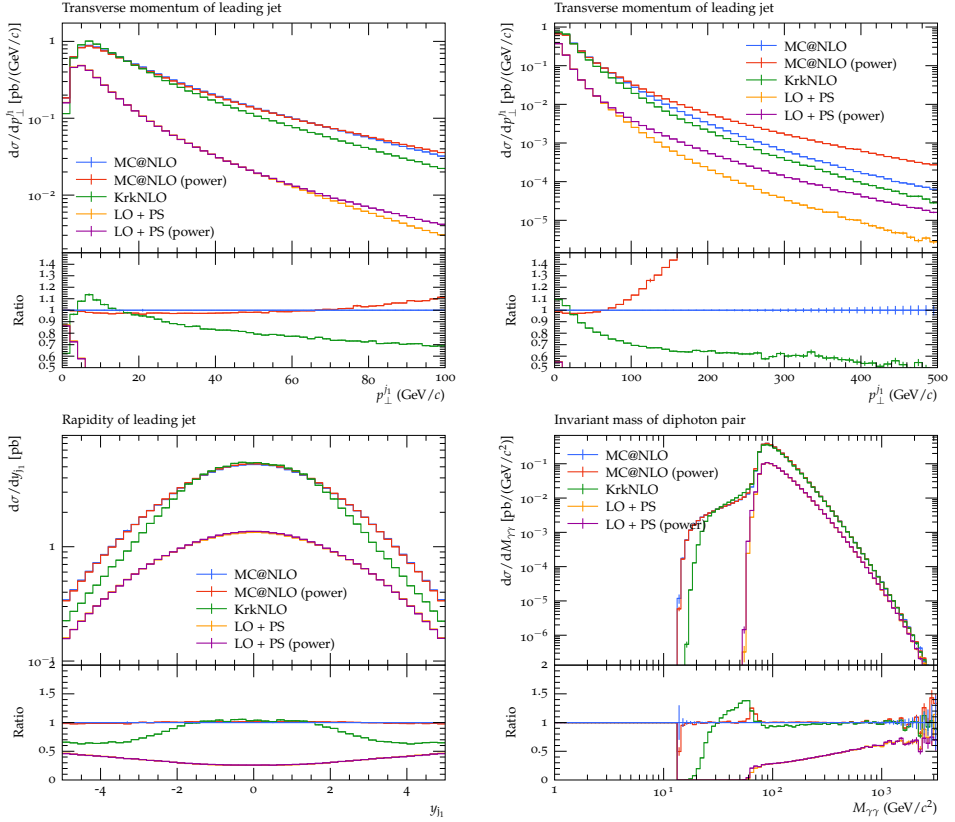


Fig. 2. ‘Full shower’ comparison of KrkNLO with MC@NLO and distributions related to the jet momenta or momentum of the diphoton system generated by the parton shower when combined with a leading-order calculation.

MC@NLO for  $p_{\text{T}}^j \gtrsim 150$  GeV. The  $d\sigma/dM_{\gamma\gamma}$  distribution (*cf.* Fig. 2) shows generally good agreement between the methods where they have NLO accuracy, with a region of disagreement arising below the boundary of the Born phase space at  $M_{\gamma\gamma} = 80$  GeV in which the two predictions are effectively accurate to leading-order only. At the lower edge of the permissible phase space, the two photons each attain their lower-bound  $p_{\text{T}}^{\text{cut}}$  and with  $\Delta R_{\gamma\gamma} \approx 0.4$  recoil against a harder jet; the behaviour of the predictions in this region is similar to that implied by the tail of the  $d\sigma/dp_{\text{T}}^{j1}$  distribution.

The azimuthal separation of the photons  $d\sigma/d\Delta\phi_{\gamma\gamma}$  (*cf.* Fig. 3) shows reasonable agreement between the KrkNLO and MC@NLO (default) methods; interestingly, as the KrkNLO method uses an unrestricted phase space like the MC@NLO ‘power’ shower, the resulting shape is retained but the normalisation remains consistent, to within 10–20%, with (default) MC@NLO.

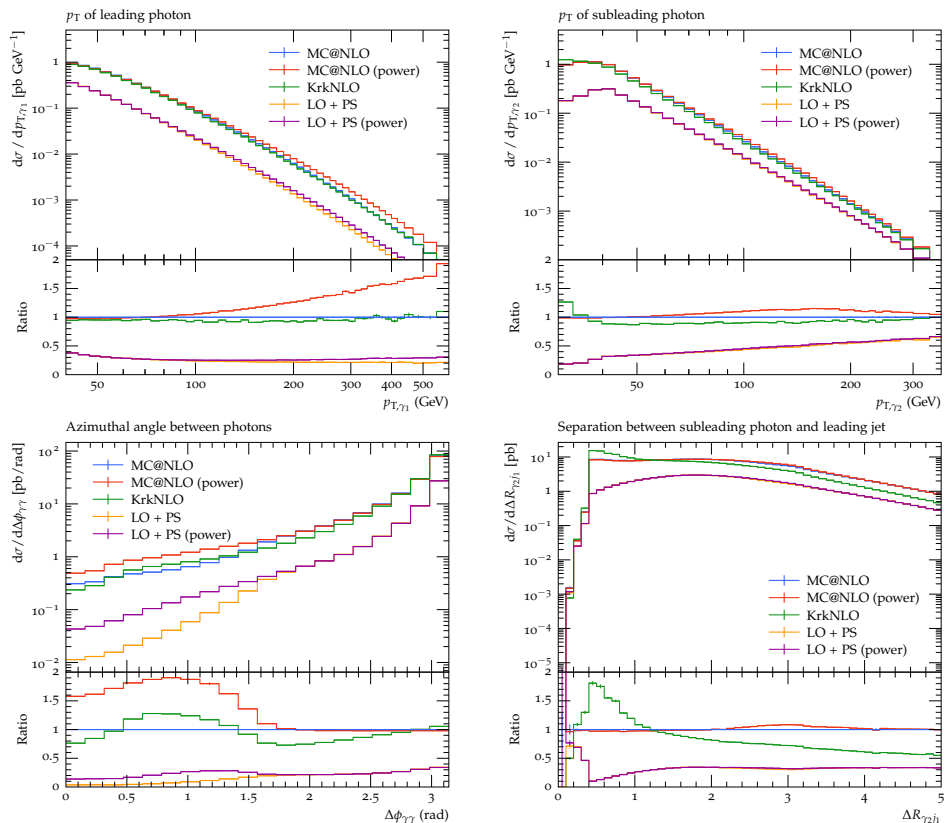


Fig. 3. ‘Full shower’ comparison of KrkNLO with MC@NLO in both ‘default’ and ‘power’ shower configurations, and the photon distributions generated by the parton shower when combined with a leading-order calculation.

The comparison of differential cross sections as predicted by these methods, and compared where possible with ATLAS data, is performed in further detail in [8]. However, for processes such as diphoton production, the opening of new partonic channels at NNLO (as well as the phase space available to soft gluons due to asymmetric photon cuts) makes NNLO corrections [32–35] vital for the accurate description of experimental data, which is therefore unattainable by any NLO matching method. Assessments of the significance of the choice of matching method and resulting uncertainties for LHC phenomenology based on comparisons to experimental data must therefore await the further development of the KrkNLO method for other processes.

## 5. Conclusions

In these proceedings, we have given a brief account of the KrkNLO method for the matching of NLO fixed-order calculations to parton showers. We have outlined its implementation in *Herwig 7* and presented and summarised results illustrating the impact of the choice of matching scheme, and the higher-order corrections implicit in the choice of any single scheme, upon the phenomenology of the predictions generated for diphoton production at the LHC. A more detailed account is available in [8].

We anticipate that code allowing the calculation within the KrkNLO method of predictions for further colour-singlet processes will be made available in a future public release of *Herwig 7*, as a result of which it will be possible to further study the significance of the choice of matching scheme for LHC physics.

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