Various types of theoretical uncertainties by example of the elastic nucleon-deuteron scattering spin correlation coefficient $C_{z,x}$

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Abstract. The elastic nucleon-deuteron scattering process can be used to study three-nucleon dynamics, but in order to draw reliable conclusions based on the comparison of theoretical predictions and data, an estimation of theoretical uncertainties is necessary. We consider the statistical errors arising from a propagation of uncertainties of the two-nucleon interaction parameters to the three-nucleon system, truncation errors, regulator dependencies, and uncertainties arising from using the formalism of partial waves. We give an example of the nucleon-deuteron spin correlation coefficient $C_{z,x}$, obtained within the Faddeev approach, to discuss magnitudes of the above mentioned theoretical uncertainties. We find that using various models of the nuclear forces results in the dominant theoretical uncertainty for the nucleon-deuteron elastic scattering observables.

1 Introduction - problem and solution method

The necessity of reliable theoretical errors estimation steams from the growing precision of experimental data as well as from the fact that nowadays we are aiming to study details of the underlying physics. Investigation of the nuclear interaction within the elastic nucleon-deuteron (Nd) scattering is a good example: while the two-nucleon (2N) potential is relatively well-known, the details of the three-nucleon (3N) force are still not so clear. We refer the reader to our recent work [1] for more general discussion and in this contribution we only briefly summarize these studies. The possible sources of theoretical uncertainties in the few-nucleon sector are the statistical errors arising from a propagation of uncertainties of free parameters of the 2N interaction to the 3N system, the truncation errors related to neglecting the higher orders of chiral expansions, the dependence on regulator functions, the numerical uncertainties as well as the uncertainties bound to the computational scheme used, and last but not least the uncertainty arising from using the various models of nuclear interaction. It will be shown that the latter one is a dominant source of uncertainties for predictions on the three-nucleon scattering observables.

We are especially interested in the statistical errors, i.e. uncertainties of 3N observables which arise from the propagation of uncertainties of free parameters of a 2N interaction. They in turn arise from uncertainties of 2N data used to fix model parameters. Investigation of these uncertainties has become possible recently with the availability of models of nuclear

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forces for which free parameters are fixed by performing a careful statistical analysis [2–4]. We use the One-Pion-Exchange (OPE) Gaussian 2N potential derived by the Granada group [3]. The structure of the OPE-Gaussian force is similar to the structure of the standard semi-phenomenological AV18 model [5]. The OPE-Gaussian potential $V(\vec{r})$ consists of the long-range $V_{long}(\vec{r})$ and the short-range $V_{short}(\vec{r})$ parts. The $V_{long}(\vec{r})$ part is the OPE force supplemented by electromagnetic corrections. The $V_{short}(\vec{r})$ component is built from 18 operators $\hat{O}_n$, among which 16 are the same as in the AV18 model. Each of them is multiplied by a linear combination of the Gaussian functions $F_k(r) = \exp(-r^2/(2a_k^2))$, with $a_k = \frac{a}{1+k}$, and the strength coefficients $V_{k,n}$:

$$V_{short}(\vec{r}) = \sum_{n=1}^{18} \hat{O}_n \left[\sum_{k=1}^{4} V_{k,n} F_k(r) \right]. \tag{1}$$

The important feature of the OPE-Gaussian potential is that not only the values of its parameters $(a, V_{k,n})$ but also their covariance matrix are known. This allows us to use a statistical approach to estimate some theoretical uncertainties. Namely, given the expectation values and correlation parameters for the potential parameters, we sample (from the multivariate normal distribution) 50 sets of the potential parameters. For each set we calculate within the Faddeev approach [6] the 3N observables, what allows us to study the statistical properties of the obtained predictions. Especially the spread of these predictions is a measure of the magnitude of the statistical error, see Ref. [1] for discussion of various estimators of this dispersion.

Working with models of nuclear potentials derived within the χEFT approach, see e.g. [7], the truncation errors for observables occur. A simple prescription on how to estimate them has been proposed in Ref. [8] for the 2N system and extended to many-nucleon systems (if many-nucleon forces are neglected) in Refs. [9, 10]. This prescription is based on the assumption that the contribution to the observable from consecutive chiral orders preserves similar perturbative pattern as the chiral force itself. In case of calculations beyond N^{2}LO a correction for neglecting the 3N interaction is introduced. We also use this method for our specific case of 3N observables.

Regulator dependence is one more source of uncertainty of chiral predictions. We regard two models of the chiral interaction at the fifth order of the chiral expansion (N^{4}LO): one with the semi-local regularization in coordinate space [7] and one using the nonlocal regularization in momentum space [11]. In both cases the regulator function depends on one parameter whose range has been proposed by the authors of Refs. [7] and [11]. The 2N, 3N, ... observables show a dependence on the value of the regulator parameter used and the spread of the predictions due to the different values of parameters measures the related theoretical uncertainty.

Other types of theoretical uncertainties are bound to specific approaches and computational methods. In our case the main contribution to such uncertainties comes from using the formalism of partial waves and neglecting states with high orbital momenta.

To calculate 3N observables we use the formalism of Faddeev equations which is a standard technique used to investigate nucleon-deuteron scattering. This framework as well as our numerical performance is described in detail e.g. in [6]. In the present work, we neglect the 3N interaction and apply only the two-body force, which enters the Faddeev equation via the t-matrix operator. That operator, the free 3N propagator and the permutation operator are used to obtain the transition amplitude for elastic Nd scattering, from which any observable for this process can be calculated. As mentioned above our numerical solution is obtained by using 3N partial waves, and we take into account all states with the two-body angular momentum j up to $j_{max} = 5$ and the three-body total angular momentum J up to $J = \frac{25}{2}$.

2 Results for nucleon-deuteron spin correlation coefficient

We have chosen the spin correlation coefficient $C_{z,x}$ to exemplify magnitudes of uncertainties discussed in Section 1. In Fig. 1 we show this observable at three incoming nucleon laboratory energies: E=13, 65 and 200 MeV. At the two lower energies predictions based on the AV18 and on the OPE-Gaussian (with the central values of parameters) are very close to one another. Also the red band, which comprises 34 predictions (what corresponds to $\Delta_{68\%}$ estimator, see [1]) based on the sampled sets of OPE-Gaussian force parameters practically collapses to the line. This means that the statistical uncertainty of the $C_{z,x}$ is very small for all scattering angles. At the highest energy the spread of the OPE-Gaussian based results becomes somewhat bigger, however, it remains small compared to the difference between predictions based on the AV18 interaction and on the OPE-Gaussian force.

In Fig. 2 we show the estimators for various theoretical errors for the spin correlation coefficient $C_{z,x}$ at E=65 MeV and at the scattering angle $\theta_{c.m.} = 120°$. The vertical lines represent predictions obtained with various models of the nuclear interaction and thus their spread estimates one type of theoretical error. Further, the horizontal lines show magnitudes of the remaining theoretical uncertainties. The red solid horizontal line shows the range of the predictions obtained with the 34 sets of parameters of the OPE-Gaussian force. The magenta line and the green solid lines show the predictions obtained with the chiral N^{4}LO forces in the full range of proposed regulator values from 0.8 to 1.2 fm for the Bochum-Bonn model and with the cutoff in the range from 450 to 550 MeV for the Idaho-Salamanca interaction, respectively. The truncation error for the Idaho-Salamanca N^{4}LO force at Λ=500 MeV is relatively small compared to the truncation error for the Bochum-Bonn N^{4}LO potential at R=0.9 fm. Last but not least the orange line show the difference between predictions based on all partial waves with j=4 and j=5. Note, that due to a well confirmed good convergence of our numerical scheme [6], the actual uncertainty related to the numerical performance (difference between predictions with j=5 and the limit of infinite number of partial waves) is smaller then the range shown by the orange line.
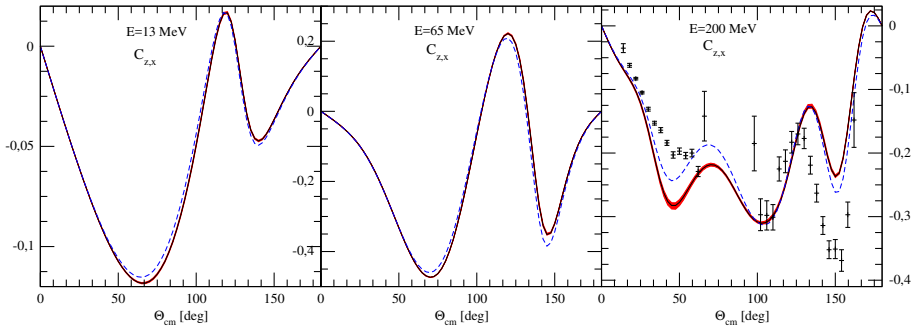

Figure 1: The Nd elastic scattering spin correlation coefficient $C_{z,x}$ at the incoming nucleon laboratory energy a) E=13 MeV, b) E=65 MeV, and c) E=200 MeV as a function of the c.m. scattering angle $\theta_{c.m.}$. The black curve represents predictions obtained with the central values of the OPE-Gaussian parameters, the red band shows statistical uncertainty, and the blue dashed curve represents predictions based on the AV18 force. The data at E=200 are from [12].

3 Summary

Summarizing, we applied the OPE-Gaussian force to study the propagation of uncertainties of 2N interaction parameters to 3N observables. We compared this uncertainty to the other types of theoretical errors present in the Nd elastic scattering analysis. We conclude that
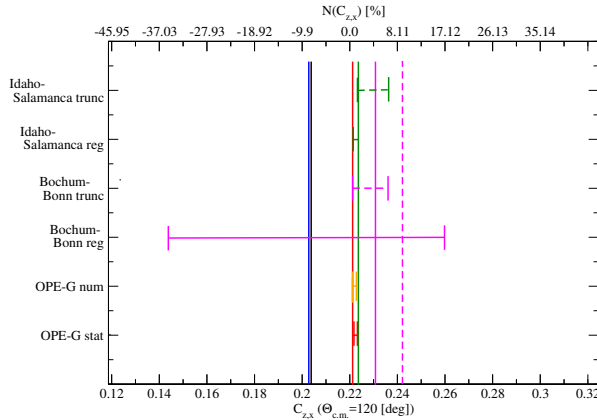
Figure 2: The *Nd* elastic scattering spin correlation coefficient $C_{z,x}$ and its various theoretical uncertainties at E=65 MeV and at scattering angle $\theta_{c.m.} = 120°$. The x-axis at the bottom shows the values of $C_{z,x}$, the x-axis at the top shows the relative difference of predictions with respect to the OPE-Gaussian results. The vertical lines show the position of $C_{z,x}$ obtained with the AV18 force (black line), the OPE-Gaussian force (red line), the Idaho-Salamanca N^{4}LO Λ = 500 MeV force (green line), the CD-Bonn (blue line), the Bochum-Bonn N^{4}LO R=0.9 fm force (magenta solid line), and the Bochum-Bonn N^{4}LO R=1.0 fm force (magenta dashed line). The horizontal lines represent (from the bottom) the statistical error (red line), the difference between the OPE-Gaussian predictions with j_{max} = 5 and j_{max} = 4 (orange line), the regulator dependence for the Bochum-Bonn force (magenta solid line), the truncation error for the Bochum-Bonn force (magenta dashed line), the regulator dependence for the Idaho-Salamanca force (green solid line), and, at the top, the truncation error for the Idaho-Salamanca (green dashed line), see text for details.

for the spin correlation coefficient $C_{z,x}$ the resulting statistical uncertainty is smaller than the dependence on interaction models both at low and at high energies. Also, we can conclude that the dominant uncertainty arises from using various models of the nuclear interaction and that the regulator dependence for the chiral models is relatively strong. These conclusions agree with results of a more extended analysis given in [1].

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