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On individuals in branching histories

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Abstract Against the background of the theory of branching space-times (BST), the paper sketches a concept of individuals. It discusses Kripkean modal intuitions concerning individuation, and, finally it addresses Lewis's objections to branching individuals.

Keywords Individuals · Modality · Space-times · Branching space-times

1 Introduction

Phrases like "a branching object" or "an individual living in branching histories" invariably bring incredulous stares. The phrases evoke an image of one and the same object miraculously existing in two (or more) alternative possible histories. Since "alternative" means that the two possible histories cannot both occur, how is it that one and the same object exists in two (or more) alternative histories? Further incredulous stares result upon hearing that none of these histories is distinguished as actual. There is also a philosophical argument, thanks to David Lewis, to the effect that branching leads to silly consequences: If one construes a modal claim "Humphrey could have had six fingers on his left hand" in terms of one and the same individual living on two branching histories, it follows (seemingly) that he has five and six fingers on his left hand.

In this paper I will argue against those incredulous stares. The rule of this game is to first acknowledge various controversies concerning objects that are independent of modeling them in a branching framework. Intuitively, individuals are persons as well as things (i.e, glasses, planets, bushes, or grains of sand, etc.) The concept is not razor

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sharp: we may have, for instance, doubts about whether ocean waves, neutrinos, or nations constitute individuals. A different host of problems is brought by biological organisms like a gigantic fungus or coral polyps or an Argentinian ant mega-colony—cf. Wilson (1999). Next, there is a question of whether individuals can have a discontinued existence, as stories abound about watches or tents that were first taken apart and then put together. To finish this list of controversies, since special relativity makes the spacial size frame relative, there is a problem of squaring a concept of spatially thick objects with special relativity.

We will not concern ourselves with these controversies. We will rather assume that there is some concept of individuals before branching enters the stage. We then argue that first one needs branching individuals and, second, give and discuss a definition of branching individuals against the background of branching space-times (BST) of Belnap (1992). Let us, however, begin with a motivating story.

After over four decades of work at the University in Jena, Gottlob Frege retired in 1917 and moved to Bad Kleinen, a small town in Pomerania. The reasons behind this relocation are not completely clear. On the one hand, incentives in favor of the move included the rising costs of living and the uncertainty of life in Jena during the First World War, sentimental family ties to Pomerania, and the peace and quiet and favorable climate offered by Bad Kleinen—all of which would be important for the frail health of the Jena philosopher. On the other hand, however, the move meant severing ties with the university circle and its environs. It also entailed a weakening of contact with Jena scholars, an end to frequent visits to the university library, and even to a change in various small but significant rituals (i.e., the afternoon apple strudel at Baumann's across from the Paradise station). At first, Frege pushed aside the thought of moving. It appears that only a visit from a cousin of his mother, Franz Bialloblotzki in the early spring of 1917 was decisive.[1] One should believe that—prior to Bialloblotzki's visit—it was really possible for Frege to spend his retirement in Jena. In contrast, in 1917 it was not really possible that Frege would purchase a Trabant some time later (Trabants were produced in the German Democratic Republic between 1957 and 1991 at the VEB Sachsenring Automobilwerke in Zwickau near Jena).

How are we to think of the real (also known as historical) possibilities which (some) individuals possess at some instants in time? One should perhaps begin by asking how individuals should be represented within the framework of a modal discourse. Modal discourse is studied via modal logic, so it is natural to ask how individuals are represented therein. Digging into this matter, we very quickly notice that the structures investigated by the logician are simple in comparison with those required by the metaphysician. Introducing a set of possible worlds, the logician often writes that it is merely a set of indices by which he presumably means that possible worlds are simple entities—i.e., not analyzable further by the logician. In contrast to the logician's approach, the possible worlds of the metaphysician are inhabited by events, processes, and individuals, some of which possess a temporal and/or spatial structure. To the query as to "how to represent individuals" the modal logician usually responds: "I postulate a domain of quantification and assume that it comprises individuals". The second half

[1] This story contains some fictitious elements.

of this answer is, however, controversial at present—for logicians as well. Why should one assume that individuals are to belong to a domain of the quantification? Instead of speaking of individuals, we could speak of substances, or just things that exist—here referring to such a category of objects as exists in the most irreducible manner. Why then, in the process of building a theory and having some pre-theoretical assumptions that such-and-such things actually exist, should we have to ensure that these things belong to the domain of quantification of the theory under construction? In fact, that this is the way one should work is expressed in Quine's (1953, p. 13) famous dictum: "[…] a theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true." But, although a burden of proof clearly lies on a proponent of the above dictum, Quine did not (as far as I know) provide any reasons why this postulate should be accepted. Moreover, there are formally precise concepts currently present in logic which do not respect Quine's requirement.

Permit an illustration drawn from temporal logic and considered in a similar context by Garson (2006, p. 290). In a temporal logic, temporal slices of individuals belong to the domain of quantification. Applying Quine's postulate to temporal logic, we are thus committed to the existence of temporal slices. Still, looking at this intuitively, such slices are rather abstract constructions which exist, if at all, thanks to the existence of the temporal individuals of which they are slices. Temporal slices of a substance are (arguably) inventions of our mind. In the set theoretical construction of semantic models for temporal logic, however, temporal slices play a distinctive role for the logician: they constitute a domain. Nevertheless, this does not mean that they should also play such a distinctive role in ontology: there are no reasons to accept that elements of the domain stand for what is truly real.

In contrast to associating substances with elements of the domain of quantification, one might want to identify substances with some specific functions from the set of moments to the set of temporal slices, that is, to the domain. Such a function codifies what the temporal slices of a substance are at any moment in time. This concept is elaborated in Garson (2006) whose idea of representing substances (individuals) in modal logic consists in associating some function from set W of possible worlds to domain D with each substance. Thus, the set of substances is represented by a set I of some specific functions from W to D (the minimum requirement for *specific* functions is that I is not a set of *all* functions from W to D). Keeping this distinction in mind—an object is an element of a domain while a substance is a specific function—let us scrutinize Garson's argument on behalf of the modal aspect of substances:

> A similar argument may be made in the case of the logic of necessity, though it is bound to be more controversial. If we have to choose between what we call objects in the semantics, and substances, I would think it is substances that correspond to what is truly real. In the temporal case, we insisted that real things have a temporal dimension: they are not mere slices. I think that real things also have a modal dimension. A real thing would be less real if its possibilities were not included. A chair is what it is partly because it could not possibly be a desk. Part of what makes the chair what it is is that it ceases to exist through radical change (for example, if its wood were used to build a desk). I claim that for

> something to be real, it must have a 'modal history' as well as temporal one. In the same way that certain properties such as changing do not apply to time slices, the notion of what is possible for a thing does not apply to members of D, the things we paradoxically call possible objects. The members of D are modally bare particulars, in the sense that though they may have actual properties, it makes no sense to talk of what is possible or not possible for them. On the other hand, the members of I, the substances, have a modal history, which reflects the nature of things we take to be truly real. (Garson 2006, p. 293)

The argument presented above can be reinforced by pointing not to a chair, but to things about which we tend to believe that real possibilities stand open before them. Say, for example, that I am an indeterminist gardener: I believe that at a specific moment a given tree holds alternative open possibilities. For instance, it could have pushed out some branches there and then, but this possibility was not realized. Cutting down an old tree, one could take an interest in its history: when and what happened in its lifetime? Yet one could pose yet another, more difficult question: when and what possibilities stood open before this tree? The semantic conception described by Garson the logician prompts us to formulate an analogous concept of individuals in ontology. A characteristic trait of these individuals is their many-worldly aspect.

Apart from being interested in alternative possibilities that have been open, or are now open, or will be open, are there any data that call for a modal concept of individuals? That is, are there some data suggesting that the concept of spatial and temporal individuals—i.e., persisting in the actual world in space and time—is insufficient? As a matter of fact, we utter and assert such sentences as "I could have raised my hand a moment ago, but did not do so". On the other hand, we are convinced that various actions could not have really been taken—although perhaps plausibly and consistently we could think them occurring or even argue that they are not contradictory to the laws of nature. Further, if we have read something about quantum theory earlier, then we have statements in our repertoire (which we are apt to accept) such as "This electron can turn left in the magnetic field, but it could just as easily turn right". Therefore, our point of departure is semantic intuition—a tendency to make assertions such as "It happened this way, but it really could have been otherwise". In these sentences we assign real possibilities to concrete items: individuals, events, and processes. Such data—that is, our inclination to accept statements ascribing real possibilities to concrete items—has not been questioned by any school of modal metaphysics.

2 Individuals in Lewis' theory

I will begin with David Lewis' popular concept of possible worlds and his conception of individuals. Putting this most briefly, the Lewisian individual is limited to a single possible world; it is part of one possible world. At most only a counterpart of a given individual can be found in other possible worlds, never the exact same individual (Lewis 1986a, p. 206). This idea can seem radically at odds with the semantic concept described above according to which individuals possess a many-worldly aspect, at least in the sense that they are associated with specific functions defined on the set of

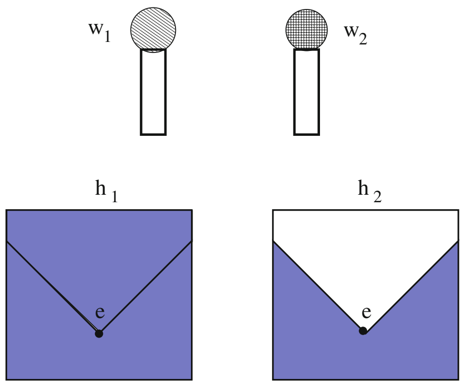


Fig. 1 At the top Lewisian divergent worlds w_1 and w_2 are represented; the *lower rectangles* indicate (similar) initial segments while the *spheres* above (in various patterns) symbolize dissimilar subsequent segments. Below, the branching out histories in branching space-times (BST) are represented here by the *squares* h_1 and h_2. The *unshaded area* indicates where the second history does not overlap with the first

possible worlds. Note, however, that this semantic idea could be realized in various ways in ontology. Arguably, Lewis realizes this idea when, on the one hand, he associates an individual name with its actual bearer in the actual world, as well as, on the other hand, to the bearer's counterpart in each possible world (and to the empty set if a possible world does not contain such a counterpart).

Be that as it may, Lewis' stance stems from his claim according to which two possible worlds do not share any parts (their intersection is empty), but can have some qualitatively similar (with respect to some properties) segments. Two worlds whose initial segments are qualitatively similar with respect to some specific relations, but whose subsequent segments are not qualitatively similar with respect to these relations are labeled "divergent worlds" by Lewis. In contrast to this concept, branching theories assume that any two possible worlds (known as histories in these theories) have an intersection which is non-empty (see Fig. 1). The Lewisian point of view leads to counterintuitive consequences which have been described over the past two decades. One of the charges pertains to the question of how my counterparts, inhabiting other possible worlds, are key to my own vacillations, hesitations, worries, etc. which are expressed in modal language? This is the essence of Kripke (1980) objection leveled against Lewis (see footnote 13 to Lecture I): on the basis of Lewisian theory, when we state that Humphrey could have won the presidential elections, then we are speaking of his counterpart and not Humphrey himself. Illustrating this further, about an hour ago I was nearly struck by a car. Hence I say, "I could have been run over, but fortunately I dodged out of the way". Am I really rejoicing because in another possible world my twin was run over, but in this one I was saved? Clearly not.[2]

At least as serious is another allegation regarding the relation between events and individuals. Lewisian events, especially those among which causal relationships appear, are many-worldly objects. For example, my morning stumble on the stairs has its parts in other possible worlds because I could, after all, have stumbled a bit earlier

[2] Lewis (1986a) responded to Kripke's objections, the inadequacy of this reply has been argued for by McGlone (2008). But for a recent defense of Lewis's view, cf. Borge (2006).

or later.[3] Presumably we would qualify these cases as a single event (my morning stumble on the stairs). Yet, paradoxically, my stumbling this morning did not involve me, but rather my other-worldly clones. The agent of my stumbling is not me as my stumbling is executed by my counterparts.

At this point, I will interrupt this discourse with Lewis, referring the reader to Belnap et al. (2001) or Placek (2001). I will return to Lewis in Sect. 2 in order to respond to his objections against branching histories and branching individuals. In order to present the latter concept, a bit of information on branching-type theories is necessary.

3 Individuals in branching-type theories

3.1 Branching theories

The starting point for branching-type theories is an intuition regarding real possibility: some events have more than one possible continuation. About such an event we may say (tenselessly) that it faces real alternative future possibilities. Hence the concept of real possibility is relative to the event itself; with the exception of specific cases, a real possibility open in one event is not an open possibility in another.[4] A set of possible events, delineating which possibilities are open to which events, is what we call "Our World".

Branching theories aim at two goals: to offer a general metaphysical model of our indeterministic world, as well as to offer a semantics for languages with historical modalities, tenses, and indexicals. Certain structures in a model of a branching-type theory are interpreted as a multiplicity of the real possibilities open at an event.[5] Branching models, understood as semantic models, assess as true some statements of the following form:"It is possible that A and it is possible that $\neg A$" or "Once it was possible that A, but now it is impossible that A".

In branching-time (BT) theory, events are understood as momentary time-slices of the universe. In other words, a BT event is the class of all commonsense point events simultaneous with a given point event. (This reference to the simultaneity means that BT is not a relativistic theory). A BT model is a non-empty partially-ordered set of possible point events, satisfying the requirement of no backward branching. The ordering is interpreted as "one event lies in the past of another". Possible histories (in some sense reminiscent to the Lewisian possible worlds) are defined as maximal chains in the base set. In contrast to Lewis, however, BT postulates that the intersection of any two histories is non-empty. In addition to dealing with events, the theory allows us to speak of the instants (moments) at which they occur. The first published (informal)

3 I refer here to so-called non-fragile events of Lewis (1986b). Lewis allows for events limited to a single possible world (so-called fragile), but these are not, according to him, relata of causal relations.

4 In order for this particular case to occur, two events must be upper bounded, be contained within the same histories, and have the same set of maximal elements.

5 The structures in question are some specific forks composed of maximal chains in a base model, which Placek and Belnap (2010) called "modal forks". This publication proves some facts concerning modal forks as well.

presentation of this theory is found in Prior (1967). Nonetheless, its unpublished formulation is already found in an earlier (1958) letter from Kripke to Prior. It is, however, to Thomason (1970) that we owe the formally rigorous presentation of branching-time theory (for more on this, see Appendix).

In comparison with BT, the branching space-times (BST) theory of Belnap (1992) permits one to work with the concept of a point event and accommodate some rudiments of special relativity. Further, in contrast to BT, it allows for incomparable events belonging to one history: neither belongs to the absolute past, or future of the other. As for a BST model, it is defined as a non-empty partially ordered set of possible point events which meets certain axioms (listed in Appendix here). The ordering is required to be dense and is interpreted as "one event lies in a possible future of the other". Histories (defined as maximal upward-directed subsets of a base set) are supposed to be "similar" to Minkowski spacetime. Yet, while histories are isomorphic to Minkowski spacetime in some specific BST models known as Minkowskian Branching Structures and elaborated by Müller (2002); Wroński and Placek (2009), and Placek and Belnap (2010), there are BST models in which this is not true. Two incomparable events belonging to some single history are said to be space-like related (SLR). It is postulated that the intersection of any two histories is non-empty. In addition to events, in some (but not in all) BST models one can define spatiotemporal locations (geometrical points) at which point events occur (see Appendix).

3.2 Branching individuals

Before embarking on a formulation of the concept of the branching individual, I will introduce two caveats. Firstly, I will not put forward a sufficient, but rather a necessary condition—that is, one of the form "If something is an individual, then it possesses such-and-such traits". A sufficient condition amounts to a statement of a principle of individuation, and this is an extraordinarily difficult task. Next I assume a simplification according to which individuals are spatially non-extended. It is my belief that this constraint does not crucially impinge on the following construction. In what follows, I give a sketch of how to generalize the approach described here to individuals which are spatially extended as well as to those which alter their spatial extension (thickness) in time. This requires sharpening of our frequently fuzzy intuitions concerning the spatial aspects of individuals, which amounts to addressing questions like:

1. Where exactly does my body end in space?
2. Are the empty spaces inside my body part of me?
3. Is my thickness to be understood as frame-dependent, and if so, is there a preferred frame of reference with respect to which my thickness is to be assessed?

Let us rehearse the required notions—against the background of a nonempty partially ordered set $\mathcal{W} = \langle W, \leqslant\rangle$. We write $x < y$ iff $x \leqslant y$ and $x \neq y$. A chain in $\mathcal{W}$ is a subset of W any two elements of which are comparable by $\leqslant$, i.e., $I \subseteq W$ is a chain iff $\forall x, y \in I : x \leqslant y \vee y \leqslant x$. We say that I is a maximal chain in W if for every B such that $I \subsetneq B \subseteq W$, B is not a chain in W. And, if I is a lower and upper bounded subset of a maximal chain M that contains all the elements of M between the least upper bound and the greatest lower bound of I, we say that I is an interval.[6]

In the definition below, a set A is identified with an individual whereas maximal chains in A are identified with possible lives of this individual—exactly like in the postulates 1–4 of Belnap (2011). In contrast to Belnap's concept, however, we do not assume that an individual must have a beginning or end to its life, however. We allow for, say, an elementary particle living for ever.

Definition 1 If A is an individual in a model $\mathcal{W} = \langle W, \leqslant \rangle$ of BST, then

1. $A \subseteq W$;
2. if $x, y \in A$ and $x < y$, then there is an interval $I \subseteq A$ such that $x, y \in I$;
3. if $x, y \in A$ and $x, y \in h$ where h is a history in $\mathcal{W}$, then either $x < y$ or $y \leqslant x$,

Let us take a closer look at each of the above-listed conditions. The first point requires that individuals are spatiotemporal in the sense that they are composed of, or one can differentiate in them, some spatiotemporal events. From a later perspective, we can say about some individual that some of its/his/her events belonged to once open real possibilities, which however have not been realized. Thus (saying this again from a later perspective), an individual consists of what was in its/his/her life, as well as of what was once possible for it/him/her.

With clause (2) we decided against discontinued existence: the clause is to capture an intuition that in every possible life of an individual there are no breaks or gaps. This can be read as a postulate of causal continuity of the stuff constituting an individual: its later phase is influenced by its earlier phases. Or, assuming that the model permits the introduction of instants (BT) or spatiotemporal locations (BST), we may read this condition as: "There are no gaps in the lifetime of the individual" (BT); or "There are no gaps in the (spatiotemporal) world line of the individual".

Finally, clause (3) expresses our simplifying premise that individuals are spatially non-extended. Counterposing the clause, if two incomparable events belong to an individual, then there is no history to which both of them belong. The clauses of the definition entail that no history contains two maximal chains in A.[7]

The simplicity of the above definition may erroneously suggest that this definition would deliver an adequate concept of individuals if applied against the framework of branching time. The problem is that in BT an interval is a spatially maximal object—a segment of history, which makes every individual maximally thick.[8]

3.3 Individuals in BST

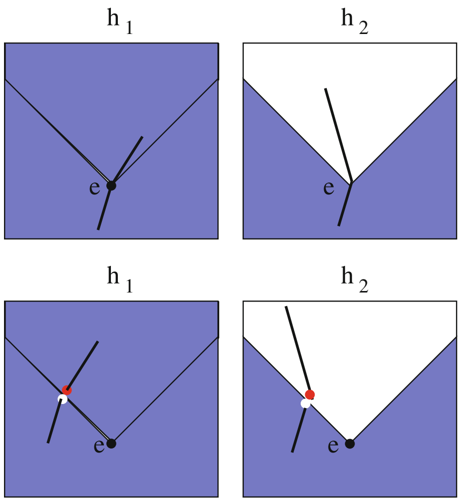
Figure 2 presents two models—upper and lower—each with two histories. In both of the models, the individual has its beginning in a single spatiotemporal event, and

6 The existence of infima of lower bounded chains and history-relative suprema of upper bounded chains is guaranteed by BST axioms—see Appendix.

7 For a simplification of this definition I owe thanks to Tomasz Bigaj.

8 I made this error in an earlier version of this paper, and it was pointed to me by N. Belnap and his seminar group.

Fig. 2 Individuals in BST. Above the individual has a choice; below we see indeterminism without a choice



later has two alternative future possibilities. The use of the possessive "has" turns our attention to the difference between the models, however. In the first one (the upper one), the choice event e (i.e., the maximal element in the intersection of two histories $h_1 \cap h_2$) belongs to the individual. Because we have assumed that objects are spatially non-extended, it is natural to conclude that this event represents a choice of our individual (more precisely, a last event of the individual's indecision).

In contrast to this situation, the choice event e in the lower model of Fig. 2 is external to the individual. This is due to the fact that the part of the individual contained in the intersection $h_1 \cap h_2$ does not possess a maximal element. Accordingly, there is no choice event which would belong to this individual.[9] This corresponds to the situation in which a choice event—say the toss of a coin—is space-like related to some part of the life of the individual. The coin lands either heads or tails and—since both these possibilities are open in the event of a toss-up—the individual can subsequently live either in the scenario in which it was tails, or in the scenario in which it was heads. Since the individual does not have a choice here, although each of the two alternative scenarios is possible for him/her/it—"indeterminism without choice" is an apt label for this phenomenon (cf. Belnap 1992).

Let us now look at structures which do not meet the definition of an individual. On the one hand, the structure contained in the two histories of the upper model of Fig. 3 (and drawn with thick lines) does not fulfill the second clause of our definition as its life is not continuous; on the other hand, the structure contained in the two histories of the lower model violates the third clause because it contains incomparable elements in history h_1 (and in history h_2 as well).

[9] Thanks to clause 4 in definition 4, the subset mentioned does have one supremum in history h_1 and a different supremum in history h_2, but does not have a supremum in the model.

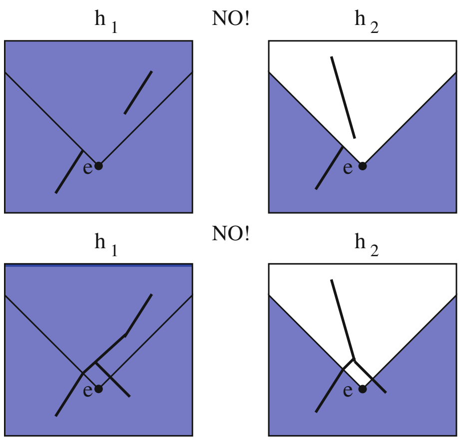


Fig. 3 Two structures, depicted with *thick lines*, one in histories h_1, h_2 of the *top* and the other in the histories h_1, h_2 of the *bottom* model are not BST individuals

3.4 Additional conditions of individuals

In this section we will discuss some additional conditions that might be added to the above-described conception of individuals. Thinking over how these conditions should be formalized, we often discover the multiplicity of their meanings.

To start with, the fact that individual object A must have come into existence is expressed as follows: each maximal chain in A is lower bounded. This can be further refined by requiring that each maximal chain in A has an infimum in A. Clearly, the meaning of this requirement is that the event of one's birth belongs to one's life. Analogically, one can formulate a thesis that individual A must die some day: each maximal chain in A is upper bounded, or (more controversially) has a supremum in A.

More interesting is the matter of representing that which we call Kripkean intuitions: we tend to accept that an individual could not have been born to parents other than those that it really had. Since BST has no machinery to represent an individual's parents, we must contend ourselves with analyzing a more sweeping claim: "Individual A came into being in such and such a manner and could not have come into being in any other way". Many instances of this sweeping claim seem patently false, however, and likely have not been believed by anyone. For instance, in our traveling times we seem to believe that we could have been born (or, conceived) in a barely different place, at another spatiotemporal point, a little bit earlier/later, in a different event.

It is still worth looking (I believe) at the claim "Individual A came into being in such and such a manner and could not have come into being in any other way" from a branching perspective, since this perspective permits us to clearly see that some intuitions above pertain to events, while others to spatiotemporal structures. From the event perspective, the fact that A could not have come to be by any other means can be expressed by saying that A has a minimal element, or (alternatively) that each maximal chain in A has an infimum in W, and for all maximal chains l_1, l_2 in A : $\inf(l_1) = \inf(l_2)$.

Moving on to the spatiotemporal perspective, let us recall that the concept "in the same place" cannot be expressed in BT because this theory overlooks the spatial aspects of the world. In turn, for relativistic reasons, in BST we cannot express the concept "in the same time" and "in the same place". In lieu of this we have in BST the concept "at the same spatiotemporal location (point)". The spatiotemporal location of an event e is then defined as the equivalence class $[e]_S$ on a base set with respect to a certain relation S (see Appendix). The thesis that "A could not happen at another spatiotemporal location" can be set down as:

Every maximal chain in $A \subseteq W$ has an infimum in W and there exists a certain spatiotemporal location $[e]_S$ to which all of these infima belong.

3.5 A note on spatially extended individuals and relativity

Special relativity entails that an individual's spatial extension is frame-relative. That is, "how thick is an individual at a given time?"—receives different answers in different frames of reference. One reaction to this observation is to distinguish a particular frame of reference for a given individual and define the individual's thickness with respect to this frame. This could be, for instance, a frame of reference in which an individual's center of mass rests. Or, in a similar vein, one may pick some particular element of an individual (say, the main cell of the brain, the center of one heart, if an individual has these), idealize it to be point-like, and consider its rest frame.[10] The success of this project hinges on the naturalness of the choice for an individual's frame of reference. Since I am skeptical about one frame of reference being more natural than another, I will sketch the other option: an individual's thickness is frame-dependent and no frame is distinguished.[11]

To put aside issues inessential for special relativistic characterization of individuals (but otherwise highly important), let us assume that an individual has no holes, that it is clear-cut where its/his/her spatial boundaries are (with respect to a given frame of reference), that it/he/she begins in one point event and its/his/her each possible end occurs at a point event. Given these assumptions,we may visualize an individual's possible life as a spatiotemporal "time-like" worm, of varying thickness, its beginning and its end being point events. To characterize such objects we need to turn to a particular class of BST models, called "Minkowskian Branching Structures" (MBS's), in which histories are isomorphic to Minkowski spacetime. For details, see Placek and Belnap (2010).[12]

To construct an MBS, one starts with a set P of physical properties (like strengths of the electromagnetic field) attributable to points in $\mathbb{R}^4$. To handle alternative

[10] A yet another option is to pick what physicists call "the rest frame of a compound object", i.e., the frame of reference in which the average momentum of the object's components is zero.

[11] For a suggestion of another option, of defining individuals in terms of causally understood transitions, see Sect. 3 of Müller (2010).

[12] The construction of MBS's harks back to Belnap's (1992) remark; MBS's were then (incorrectly) defined in Placek (2000). Müller (2002) had a correct construction, but with finististic assumptions, which later Wroński and Placek (2009) removed by accepting a (non-intuitive) postulate. Recently Placek and Belnap (2010) derived MBS's from postulates inspired by field-theoretical considerations.

possibilities, a set Σ of possible "scenarios" is postulated; an attribution function F assigns then physical properties (elements of P) to scenario-point pairs, i.e., elements of $\Sigma \times \mathbb{R}^4$. Thus, an MBS is a triple $\mathfrak{M} = \langle \Sigma, F, P \rangle$ subject to some postulates. The postulates permit one to define an equivalence relation $\equiv$ on $\Sigma \times \mathbb{R}^4$, with $\langle \sigma x \rangle \equiv \langle \eta y \rangle$ meaning: $x = y$ and x does not lie in the past-light cone of any point from which on two scenarios σ and η qualitatively diverge.

The result of this construction is that, first, elements of a base set W (which constitute Our World $\mathcal{W} = \langle W, \leqslant \rangle$) are equivalence classes $[\sigma x] := \{\langle \sigma' x' \rangle \in \Sigma \times \mathbb{R}^4 \mid \langle \sigma' x' \rangle \equiv \langle \sigma x \rangle\}$. The ordering $\leqslant$ is defined as $\langle \sigma' x' \rangle \leqslant \langle \sigma x \rangle$ iff $x' \leq x$ and $\langle \sigma' x' \rangle \equiv \langle \sigma x' \rangle$. Second, there is a one-to-one correspondence between the set of histories in $\mathcal{W}$ and the set of sets of the form $b_\sigma := \{[\sigma x] \mid x \in \mathbb{R}^4\}$, where $\sigma \in \Sigma$. And, most importantly for our task, the model has a numerical aspect, as each event $[\sigma x]$ has coordinates $x \in \mathbb{R}^4$. How should one understand these numbers? As expected, these are spatiotemporal coordinates of possible events in a given frame of reference. In contrast to special relativity, however, the coordinates are not assigned to events from one history only, but to all events that were, are, or will ever be, possible. Thus, a given MBS, with the inherent values of real numbers, depicts a world, in its spatiotemporal and modal aspects, from within a given frame of reference. How can one obtain a representation of the same world, but from another frame of reference? The answer is standard: to obtain an MBS depicting the same situation as that of an initial MBS, but from a different frame of reference, apply Lorentz transformation to the "numerical" part of the initial model. That is, if $e = [\sigma x]$ is an event as represented in an initial model associated with one frame of reference, then this event as represented in a model associated with another frame of references is $e' = [\sigma Lx]$, where L is a Lorentz transformation between the two frames. In this way, an individual as represented in a model associated with one frame of reference, is related to its representations in other models associated with different frames of reference. One can then define a (non-relative) individual as a class of frame-relative representations, every two of which are related by a certain Lorentz transformation.

Having explained frame-dependence, let us return to a question how to define a spatially extended (frame-dependent) representation of an individual in an MBS model. Before branching enters the stage, one needs to generalize the concept of time-or-light-like path, i.e, a path in $\mathbb{R}^4$, whose every two points are time-like or light-like, to a "time-like" worm or tunnel (a region in $\mathbb{R} \times \mathbb{R}^3$) with varying in time spatial extension. Given our simplifying assumptions that an individual begins at a point events and each possible end of it occurs at a point event as well, the generalization is easy to accomplish. One needs to capture a concept of a region between two time-or-light-like paths joining the top point with the bottom point. This exercise in the calculus should not concern us here: we simply assume that there is a concept generalizing time-or-light-like paths to time-like worms whose spatial extension vary in time, and that begin and end at points. We will call these regions of $\mathbb{R} \times \mathbb{R}^3$: "good worms".

Then a representation of an individual in an MBS (that is, an individual as represented in a given frame of reference) can be defined as follows:

Definition 2 (*MBS representation of an individual*) Let $\mathfrak{M} = \langle \Sigma, F, P \rangle$ be an MBS, which determines BST model $\mathcal{W} = \langle W, \leqslant \rangle$, where $W := \{[\sigma x] \mid \sigma \in \Sigma, x \in \mathbb{R}^4\}$ and the ordering $\leqslant$ is defined as $\langle \sigma' x' \rangle \leqslant \langle x', \sigma \rangle$ iff $x' \leq x$ and $\langle \sigma' x' \rangle \equiv \langle \sigma x' \rangle$.

If A represents an individual in $\mathfrak{M}$, then

1. $A \subseteq W$;
2. A has a proper infimum, i.e., $\exists e \in A \forall e' \in A\ e \leqslant e'$;
3. for every $\sigma \in \Sigma$: $b_\sigma \cap A = \emptyset$ or $b_\sigma \cap A = \{[\sigma x] \mid x \in w \subseteq \mathbb{R} \times \mathbb{R}^3\}$ for some good worm w, where $b_\sigma = \{[\sigma x] \mid x \in \mathbb{R}^4\}$ for $\sigma \in \Sigma$.

4 Lewis's objections

The concept of branching individuals is non-intuitive—at least at first glance. After all, despite the diverse possibilities which stood open before him, Frege left for Bad Kleinen and died there on 26 July 1925. Indeed, Frege did not branch himself off. If so, then what are we talking about when we utter the words "a part of Frege contained in a possibility that was not realized"? The most prominent objection with regards to branching individuals originates with Lewis (1986a, p. 199). The hero of this conundrum is Hubert Humphrey, the Vice President of the USA in 1965–1969:

> He could have had six fingers on his left hand. There is some other world that so represents him. We are supposing now that representation *de re* works by trans-world identity. So, Humphrey, who is a part of this world and here has five fingers on the left hand, is also a part of some other world and there has six fingers on his left hand. *Qua* part of this world he has five fingers, *qua* part of that world he has six. He himself—one and the same and altogether self-identical—has five fingers on the left hand, and he has not five, but six. How can this be?

How can we analyse such a concept—Hubert Humphrey had five fingers on his left hand but he could have had six? The theoretician of branching individuals would express it thusly: in one history, Hubert Humphrey has five fingers on his left hand, while in some other history he has six fingers on his left hand. Yet how is this possible?

Let us show first that Lewis' objection is not of logical nature, i.e., the concept of branching individuals does not lead to contradictions and the semantic of the branching model is shaped in such a way as to be capable of dealing with branches. However, in order to initiate an analysis of the Humphrey case in branching theory, we must, in accordance with these theories, interpret the following modal sentence: "Humphrey could have had six fingers on his left hand". We therefore assume that this assertion refers to a real possibility. In other words, at a certain embryonic stage of Humphrey's development, it was really possible that a sixth finger would grow on his left hand.

We will do an analysis in the framework of BT theory because it is simpler. The primary semantic idea, drawn on Prior, pertains to evaluation of sentences—a point of evaluation is delineated by two parameters: the event e and history h wherein we assume that $e \in h$. Event-history pairs which fulfill this condition will be noted as e/h. From a logical perspective, Lewis' objection amount to asking whether there is such an evaluation point e/h at which both the sentences are true. That is:

$e/h \models$ Hubert Humphrey (now) has five fingers on his left hand,

as well as

$e/h \models$ Hubert Humphrey (now) has six fingers on his left hand.

For simplicity's sake, we assume that there is some last event e^* as of which it is not yet predetermined that Hubert Humphrey will have five or six fingers on his left hand. That is, we assume that in history h_1 (after e^* but before some final event e_1 in his life) Humphrey has five fingers, while in h_2 (after e^* but before some final event e_2 in his life) he has six. If so, then the situation described in Lewis' conundrum obtains this interpretation:

For $e \in h_1 \cap h_2$ such that $e \leqslant e^*$, $h \in \{h_1, h_2\}$:
$e/h \models$ Hubert Humphrey (now) has neither five, nor six fingers on his left hand.
For $e \in h_1$ such that $e^* < e < e_1$:
$e/h_1 \models$ Hubert Humphrey (now) has five and not six fingers on his left hand.
For $e \in h_2$ such that $e^* < e < e_2$:
$e/h_2 \models$ Hubert Humphrey (now) has six and not five fingers on his left hand.

There is no obstacle to implement these requirements in a BT model. To state things bluntly, in constructing a semantic model for this Humphrey story, we will take care to preclude that at some valuation point e/h the two sentences "Humphrey has five fingers on his left hand" and "Humphrey has six fingers on his left hand" were true. The Lewis objection does not, therefore, demonstrate any contradiction or some other logical problem resulting from the concept of branching individuals. (For an exposition of BT semantics, see e.g. Belnap et al. 2001, Chap. 8.)

This objection does, however, touch upon a metaphysical problem: how should the image of branching individuals be understood? Lewis appears to be saying that it cannot be because it is absurd. To address this objection, we need to reflect on how BT/BST models should be understood, and draw a distinction between the external standpoint versus internal standpoint (cf. Belnap 2011).

From the external perspective, a branching model contains a plethora of possibilities, all on par, with no distinction between possible and actual. It is somewhat similar to the physicist's study of the possible evolutions of a given system; the study does not ask which of these possible evolutions the system actually travels. This is a "scientific view" or "a view from nowhere". If asked, from this standpoint, what an individual is, we get a picture of a bundle of the individual's alternative possible lives, and since the distinction "actual versus possible" is not available on the external perspective, there is no way to introduce the actual life of an individual. Further, since the external standpoint does not distinguish the past, present, or future, it is incapable of expressing things like "this option was once possible for an individual; it is now impossible, though now the individual has such-and-such alternative future possibilities". As a result, from the external standpoint, we get an alien representation of an individual. After all, Frege had one actual life, and although at his various phases he faced alternative possible options of how to carry on, later these options ceased to be possible.

To use the internal perspective, we need to ask, where a theoretician/an epistemic subject/an agent is to be located? No doubt, he/she is intertwined with a net of causally active events and processes, and is in various spatiotemporal relations to other things. A theoretician/an epistemic subject/an agent is a dweller of our world, and if our world contains real possibilities, he or she is a dweller of our branching world. So, how do I, as a theoretician located in a branching world, describe individuals, say, myself or Frege? Here is what I shall do. First, perhaps after consulting my watch and a GPS device, I turn to an adequate BST model, point at a certain event e in the model and declare “here it is me now”. That particular event fixes my actual past (with respect to this event). This involves ascertaining as settled all truths about my past. In my actual past there are also some chancy events from which on my life could have gone differently. These events point to possibilities that were once open. Such chancy events in the past of e permit one to evaluate as true at e sentences of the form: “I might have …, but I did not”. In contrast, some chancy events after e determine as true or false at e sentences of the form “It is (not) possible that I will …”. Needless to say, in order for a theoretician to ascertain such sentences, his or her knowledge concerning both actualities and real possibilities should be god-like. Still, it is knowledge from a given standpoint, being located at a certain event, and having a given past.

Importantly, all sentences of our object language need to be evaluated from the internal standpoint. To evaluate sentences like “It was the case that φ”, or “It is settled that φ”, etc., we need to locate events of utterance in the model, that is, events at which these sentences where uttered, and which are their “contexts of use”. However, there are sentences, formulated in a meta-language, that call for using a BST model from the external standpoint. To give an example, the question whether some two histories are isomorphic, requires the external perspective for its evaluation.[13]

Lewis’ objection that branching individuals appear absurd has some substance if considered from the external standpoint. This objection, however, is overturned if interpreted from the internal standpoint. For branching theory the concept of indexically-given modalities and tenses is essential.

5 In lieu of a conclusion: unfinished business

Haunted by a vague and indistinguishable hunch, I feel that even if I have persuaded the reader to accept the internal standpoint, I have not won over those persons who were not already convinced of branching individuals. I believe that a part of the problem is how to comprehend a multitude of possible histories. At the semantic level, there is no problem: histories provide values for one of the parameters composing an evaluation point for formulas— in much the same way as sequences of elements of a domain provide values for the assignment parameter in the definition of truth by satisfaction. As Belnap et al. (2001) say, just as it makes no sense to ask about a distinguished value assignment so, too, it makes no sense to ask about a distinguished history.[14]

Notwithstanding the above, how should we understand the metaphysics of a multitude of possible histories? As I see it, the problem stems from the concept of possible

[13] I owe this example to T. Müller.

[14] The concept of double-time reference further permits the analysis of speech acts in the framework of branching theories (Belnap 2001).

history: histories are simply too global. Our intuitions are better tailored to a local concept of possible continuations of events. A theory of modalities and tenses that works in terms of possible continuations rather than possible histories is developed in Placek (2011). However, what the concept of an individual is in a local theory of possible continuations—remains a task for future research.

Appendix: The axioms of BT and of BST

Definition 3 (*model of BT*) A model of BT is a pair $\langle W, \leqslant \rangle$, where $W \neq \emptyset$ and $\leqslant$ a partial ordering on W, that satisfies these conditions:

1. if $e_1 \leqslant e_3$ and $e_2 \leqslant e_3$, then ($e_1 \leqslant e_2$ or $e_2 \leqslant e_1$): no backwards branching;
2. for all $e_1, e_2 \in W$ there is e_3 such that $e_3 \leqslant e_1$ and $e_3 \leqslant e_2$: historical connection.

Histories are defined as maximal chains in $\langle W, \leqslant \rangle$.

In some models of BT one may define the concept of instants, which boils down to introducing an equivalence relation I on W that respects the ordering $\leqslant$. That is, writing $[e]_I$ for the class of events equivalent to $e \in W$ with respect to I, we require that (1) for every history h in $\langle W, \leqslant \rangle$ and every $e \in W$, the intersection $h \cap [e]_I$ contains exactly one element and (2) for all equivalence classes $[e]_I$, $[e']_I$ and all histories h_1, h_2: $[e]_I \cap h_1 = [e']_I \cap h_1$ iff $[e]_I \cap h_2 = [e']_I \cap h_2$, and analogously for $<$.

Definition 4 (*model of BST*) $\mathcal{W} = \langle W, \leqslant \rangle$, where $W \neq \emptyset$ and $\leqslant$ is a partial ordering on W is a model of BST iff

1. $\leqslant$ is dense on $\mathcal{W}$;
2. $\mathcal{W}$ has no maximal elements;
3. Every lower bounded chain in $\mathcal{W}$ has an infimum;
4. Every upper bounded chain in $\mathcal{W}$ has a supremum in every history that it is a subset of;
5. For every lower bounded chain $O \subseteq h_1/h_2$ there is $e \in W$ such that e is maximal in the intersection $h_1 \cap h_2$ and $\forall e' \in O : \; e < e'$.

Histories are defined as maximal upward-directed subsets of W. For $e_1, e_2 \in W$, we say that e_1 and e_2 are space-like related (e_1 SLR e_2) iff $\exists h \in Hist : \; e_1, e_2 \in h$ but $e_1 \nleqslant e_2$ and $e_2 \nleqslant e_1$.

In general BST does not permit defining instants (temporal moments) or spatial locations of events. In some models, however, one may introduce spatiotemporal locations, understood as spatiotemporal points at which events occur. In such models one defines the relation S, corresponding to the intuitive concept "possible events from different histories occur at the same spatiotemporal point". S should be an equivalence relation on W and satisfy these conditions:

(1) for every history h in $\mathcal{W}$ and every $e \in W$, the intersection $h \cap [e]_S$ contains exactly one element, and

(2) S respects the ordering $\leqslant$: for all equivalence classes $[e]_S$ and $[e']_S$ and all histories h_1 and h_2: $[e]_s \cap h_1 = [e']_S \cap h_1$ iff $[e]_s \cap h_2 = [e']_S \cap h_2$, and analogously for $<$ and SLR.

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