

Influence of scattering versus coherent parton branching on the k_T broadening of QCD cascades in a medium

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We studied the evolution of jets within a medium that contains both, transverse kicks as well as medium induced coherent radiation. In this framework parton branching occurs simultaneously to scatterings within the medium, leading to the interference effects that reproduce the well known BDMPS-Z emission rates and sizeable transverse momentum broadening. We examined the relative importances of transverse momentum broadening from the coherent splittings and different types of in-medium scatterings and found a clear hierarchy of the influences from different scattering effects and deflections during branchings.

*** *The European Physical Society Conference on High Energy Physics (EPS-HEP2021)*, ***

*** *26-30 July 2021* ***

*** *Online conference, jointly organized by Universität Hamburg and the research center DESY* ***

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The hot and dense medium of a quark-gluon plasma (QGP) can be recreated in ultrarelativistic heavy ion collisions. Since this medium cannot be accessed directly, due to the confinement of quarks and gluons, some more indirect methods need to be used to probe the medium. One suitable type of probe are jets, highly energetic, collimated, strongly interacting sprays of particles. The advantages of jets are that the initial jet particles are created at high energy densities, i.e. at early stages of the heavy ion collisions, interact with medium via processes of the strong interaction, yet do not thermalize, due to the high momentum and energy scales of the jet-particles involved. Possible processes of jet medium interaction are scatterings of jet-particles off medium particles as well as processes of parton emission induced by scatterings off medium particles. However, during the formation of a medium induced emission it is possible that the particles involved undergo multiple scatterings off medium particles, giving rise to interference effects. Spectra for this coherent medium induced radiations were first found (in the context of a QCD medium) by Baier, Dokshitzer, Mueller, Peigné, Schiff and independently by Zakharov (BDMPS-Z) [1–7]. Later, an effective splitting kernel for coherent medium induced radiations for gluons as jet-particles was derived by Blaizot, Dominguez, Iancu, and Mehtar-Tani (BDIM) [8] as

$$\mathcal{K}(\mathbf{Q}, z, p_0^+) = \frac{2}{p_0^+} \frac{P_{gg}(z)}{z(1-z)} \sin \left[\frac{\mathbf{Q}^2}{2k_{br}^2} \right] \exp \left[-\frac{\mathbf{Q}^2}{2k_{br}^2} \right] \quad (1)$$

with

$$\omega = xp_0^+, \quad k_{br}^2 = \sqrt{\omega_0 \hat{q}_0}, \quad \mathbf{Q} = \mathbf{k} - z\mathbf{q}, \quad \omega_0 = z(1-z)p_0^+ \quad (2)$$

and

$$\hat{q}_0 = \hat{q}f(z), \quad f(z) = 1 - z(1-z), \quad P_{gg}(z) = N_c \frac{[1 - z(1-z)]^2}{z(1-z)}, \quad (3)$$

where $p_0^+ = E$ is the energy of the initial jet particle, x the parton momentum fraction (with regard to the initial energy p_0^+), \mathbf{k} the jet-particle momentum components transverse to the jet axis, \hat{q} the average transverse momentum transfer, α_s the QCD coupling constant and N_c the number of colors. Using the above splitting kernel, together with a scattering kernel w (which will be defined further below) the following evolution equation over time t can be derived for the fragmentation functions D of jet-gluons in the medium [9, 10]

$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) = & \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z}p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, xp_0^+) D(x, \mathbf{k}, t) \right] \\ & + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t). \end{aligned} \quad (4)$$

where

$$C(\mathbf{l}) = w(\mathbf{l}) - \delta(\mathbf{l}) \int d^2 \mathbf{l}' w(\mathbf{l}'), \quad (5)$$

with the scattering kernels [9, 10]

$$w(\mathbf{l}) = \frac{16\pi^2 \alpha_s^2 N_c n}{\mathbf{l}^4}, \quad (6)$$

where n is the density of scatterers in the medium and [11]

$$w(\mathbf{l}) = \frac{g^2 m_D^2 T}{\mathbf{l}^2 (\mathbf{l}^2 + m_D^2)}, \quad (7)$$

with the Debye mass $m_D^2 = g^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6} \right)$, and $g^2 = 4\pi\alpha_s$.

With the Sudakov-factors

$$\Delta(p_0^+, t) = \exp \left(-t \left[\int_{|\mathbf{q}| > q_\downarrow} \frac{d^2 \mathbf{q}}{(2\pi)^2} \left(w(\mathbf{q}) + \alpha_s \int_0^{1-\epsilon} dz 2z \mathcal{K}(\mathbf{q}, z, p_0^+) \right) \right] \right), \quad (8)$$

where the notation $|\mathbf{q}| > q_\downarrow$ should indicate that the integration runs over all \mathbf{q} except those where $|\mathbf{q}| < q_\downarrow$, the above integro-differential evolution equation, Eq. (4), can be formulated as the following integral equation

$$\begin{aligned} D(x, \mathbf{k}, t) = & D(x, \mathbf{k}, t_0) \frac{\Delta(xp_0^+, t)}{\Delta(xp_0^+, t_0)} \\ & + \int_{t_0}^t dt' \frac{\Delta(xp_0^+, t)}{\Delta(xp_0^+, t')} \int_{|\mathbf{q}| > q_\downarrow} \frac{d^2 \mathbf{q}}{(2\pi)^2} \int_0^{1-\epsilon} dz \int \frac{d^2 \mathbf{Q}}{(2\pi)^2} \int_0^1 dy (2\pi)^2 \\ & \left[w(\mathbf{Q}) \delta^{(2)}(\mathbf{k} - (\mathbf{Q} + \mathbf{q})) \delta(x - y) + \alpha_s 2z \mathcal{K}(\mathbf{Q}, z, yp_0^+) \delta^{(2)}(\mathbf{k} - (\mathbf{Q} + z\mathbf{q})) \delta(x - zy) \right] \\ & D(y, \mathbf{q}, t'), \end{aligned} \quad (9)$$

in the simultaneous limits of $\epsilon \rightarrow 0$ and $q_\downarrow \rightarrow 0$. The formulation of the evolution equation as an integral equation allows numerical solution of Eq. (4) by a Monte-Carlo algorithm [12, 13]. A goal of the presented work [13] was to study the influences of non-collinear branchings and different types of scatterings in Eq. (4) on the broadening of the distribution of transverse momentum $k_T = |\mathbf{k}|$. To this end note that a collinear splitting kernel can be found as

$$\mathcal{K}(z) = \int d^2 \mathbf{Q} \mathcal{K}(\mathbf{Q}, z, yp_0^+) \frac{\sqrt{yp_0^+}}{2\pi\sqrt{\hat{q}}} = \frac{f(z)^{5/2}}{(z(1-z))^{3/2}}. \quad (10)$$

and a corresponding evolution equation [9, 10] can also be formulated as

$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) = & \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] \\ & + \int \frac{d^2 \mathbf{q}}{(2\pi)^2} C(\mathbf{q}) D(x, \mathbf{k} - \mathbf{q}, t), \end{aligned} \quad (11)$$

where

$$\frac{1}{t^*} = \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\hat{q}}{p_0^+}}. \quad (12)$$

Integration over the transverse momenta yields the following evolution equation

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, t) \right], \quad (13)$$

Thus, in order to study deviations from a possible gaussian broadening in transverse momenta, we can also construct the following case, where the momentum fractions x follow a fragmentation

function $D(x, t)$, that is described by Eq. (13) while the transverse momenta k_T are selected from a Gaussian distribution so that the fragmentation function $D(x, k_T, t)$ is given by

$$D(x, \mathbf{k}, t) = D(x, t) \frac{4\pi}{\langle k_\perp^2 \rangle} \exp \left[-\frac{\mathbf{k}^2}{\langle k_\perp^2 \rangle} \right], \quad (14)$$

where

$$\langle k_\perp^2 \rangle = \min \left\{ \frac{1}{2} \hat{q} t (1 + x^2), \frac{k_{br}^2(x)}{4\bar{\alpha}}, (xE)^2 \right\}, \quad k_{br}^2(x) = \sqrt{x E \hat{q}}. \quad (15)$$

In the above, it is assumed that $k_\perp^2 < \omega^2 = (xE)^2$. We will refer to this case as Gaussian approximation. The different cases of jet-medium interactions studied are

- the Gaussian approximation,
- the collinear branching case $\mathcal{K}(z)$ following Eq. (11) with scatterings given by Eq. (6),
- the collinear branching case $\mathcal{K}(z)$ following Eq. (11) with scatterings given by Eq. (7),
- the non-collinear branching case $\mathcal{K}(z, \mathbf{Q})$ following Eq. (4) without scatterings,
- the non-collinear branching case $\mathcal{K}(z, \mathbf{Q})$ following Eq. (4) with scatterings given by Eq. (6),
- the non-collinear branching case $\mathcal{K}(z, \mathbf{Q})$ following Eq. (4) with scatterings given by Eq. (7).

For the numerical studies a constant coupling constant of $\alpha_s = \frac{\pi}{10}$ was assumed and the following parameters for a medium that is invariant in time

$$\begin{aligned} \hat{q} &= 1 \text{ GeV}^2/\text{fm}, & n &= 0.243 \text{ GeV}^3, \\ m_D &= 0.993 \text{ GeV}, & p_0^+ &= 100 \text{ GeV}. \end{aligned} \quad (16)$$

Fig. 1 shows results for the distributions

$$\tilde{D}(x, k_T, t) = \int_0^{2\pi} d\phi k_T D(x, \mathbf{k}, t), \quad \text{with } k_T = ||\mathbf{k}||. \quad (17)$$

for different time-scales. It can be seen that the distributions exhibit different broadenings in k_T over time obeying the following ordering from smallest to largest broadening:

1. the Gaussian approximation,
2. the non-collinear branching case $\mathcal{K}(z, \mathbf{Q})$ following Eq. (4) without scatterings,
3. the collinear branching case $\mathcal{K}(z)$ following Eq. (11) with scatterings given by Eq. (7),
4. the non-collinear branching case $\mathcal{K}(z, \mathbf{Q})$ following Eq. (4) with scatterings given by Eq. (7),
5. the collinear branching case $\mathcal{K}(z)$ following Eq. (11) with scatterings given by Eq. (6),
6. the non-collinear branching case $\mathcal{K}(z, \mathbf{Q})$ following Eq. (4) with scatterings given by Eq. (6).

Furthermore, the k_T broadening can be also studied via its average value $\langle k_T \rangle$ for jet-particles with a given momentum fraction x . Results are shown in Fig. 2. Again, the different types of jet-medium interactions yield different broadenings in k_T over time, following the same ordering as before. As can be seen at low energy scales a common behavior can be found for large time-scales, corresponding to a approximately polynomial rise in $\langle k_T \rangle$.

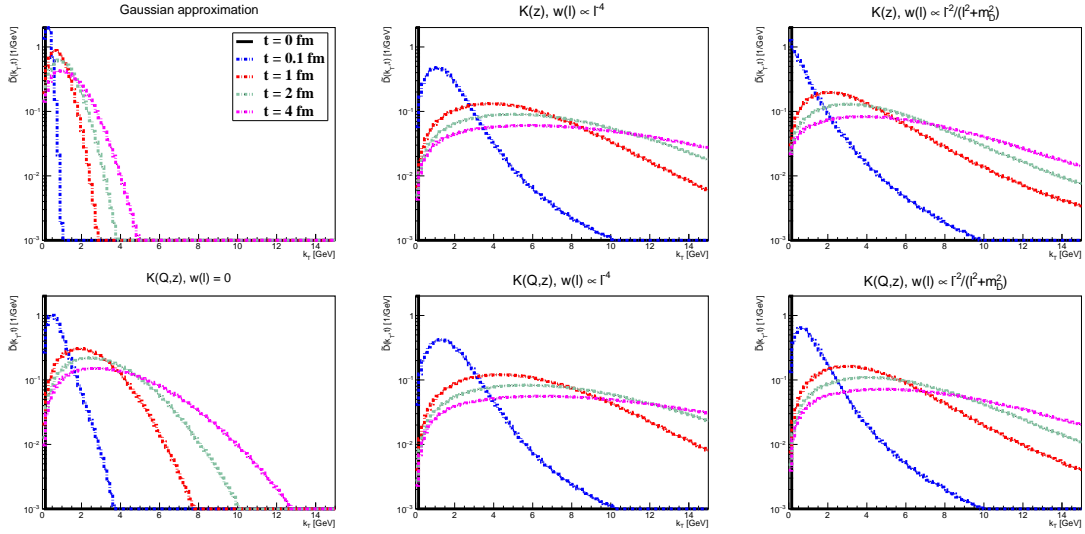


Figure 1: The k_T distributions for the evolution time values $t = 0, 0.1, 1, 2, 4$ fm, for different kernels: the Gaussian approximation, $\mathcal{K}(z)$ and $\mathcal{K}(\mathbf{Q}, z, p^+)$ (denoted as $K(z)$ and $K(\mathbf{Q}, z)$, respectively), and different collision terms: no collision term, the collision term as in Eq. (6) and as in Eq. (7).

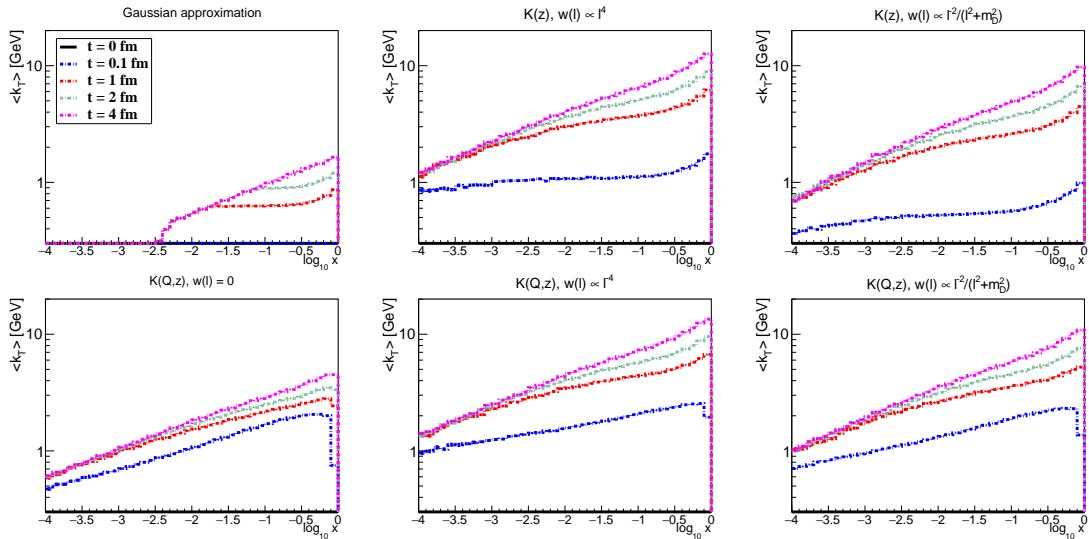


Figure 2: The $\langle k_T \rangle$ vs. $\log_{10} x$ distributions for the evolution time values $t = 0, 0.1, 1, 2, 4$ fm, for different kernels: the Gaussian approximation, $\mathcal{K}(z)$ and $\mathcal{K}(\mathbf{Q}, z, p^+)$ (denoted as $K(z)$ and $K(\mathbf{Q}, z)$, respectively), and different collision terms: no collision term, the collision term as in Eq. (6) and in Eq. (7), respectively.

1. Summary

We have studied the in-medium evolution of gluon jets via processes of coherent medium induced radiations as well as scatterings off medium particles. To this end, numerical solutions for the fragmentation functions that follow the BDIM-evolution equations Eqs. (4), (11), and (13) [9, 10], were obtained. It was found that the resulting distributions for transverse momenta k_T exhibit a clear ordering of their broadening with regard to the influences of non-collinear branchings and

scatterings off medium particles: The dominant influences on broadening come from scattering effects. Non-collinear branchings yield a non-negligible, however smaller broadening effect.

Acknowledgments

This work was partially supported by the Polish National Science Centre with the grant no. DEC-2017/27/B/ST2/01985.

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